

Prolegomena to Interdisciplinary Approach to Foundations of Science 0.

(Posing the task - Explication of the basic concepts: “Science”, “Foundations of Science”, “Interdisciplinary Approach to Foundations of Science”)

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0.1. General formulation of the task of present prolegomena

0.1.0. Preliminary note about the structure of the general formulation of the task of present prolegomena

The general formulation of the task of present prolegomena consists of following two levels:

- 1) General formulation of the task of elaboration of an interdisciplinary approach to foundations of science;
- 2) General formulation of the specification of the task of the present prolegomena to interdisciplinary approach to foundations of science.

Hence the general formulation of the task of present investigation has to be realised in two steps according to these two levels just mentioned above.

0.1.1. General formulation of the task of elaboration of an interdisciplinary approach to foundations of science

The starting point of present investigation is the following task formulated in a very general form:

- 1) to elaborate *an interdisciplinary approach to foundations of science*,
- 2) in order to be applied to elaboration of *interdisciplinary foundations of science*, as a fundamental basis for a further work on systematic (1) construction and (2) elaboration (1→2) of the various interdisciplinary scientific theories,
- 3) taking into account a preliminary, *very general, intuitive view about what is (1) the science*, and (2) *the philosophical problems of science*, (1→2) formed on the basis of experience obtained during my personal occupation until now with study in the field of some sciences as well as with treatment of their philosophical problems, and
- 4) fulfilling the requirements to an appropriate degree and form (1) of *analytical formulation of the essential propositions*, as well as (2) of their (2.1) *logical* and (2.2) *empirical* (2.1→2.2) *substantiation*.

Note 1 to section 0.1.1. The scheme of formal sentence

1) of the kind

$$“(i) \dots (i.1)\alpha_1, \dots, (i.n)\alpha_n (\rightarrow i) \beta”,$$

where $\alpha_1, \dots, \alpha_n, \beta$ are phrases and (i) is index of a phrase, means that the phrase β is a part of the whole phrase initiated by the phrase $“(i) \dots (i.1)\alpha_1, \dots, (i.n)\alpha_n”$ and concluded with the phrase $“\beta”$, so, consequently it is not a part of the whole phrase initiated by the phrase $“(i.n)\alpha_n”$; or, otherwise expressing the same, as a result of the use in the given case of the operator $“(\rightarrow i)”$ we obtain the phrase

$$“((i) \dots (i.1)\alpha_1, \dots, (i.n)\alpha_n) \beta”,$$

so, consequently not the phrase

$$“((i.n)\alpha_n) \beta”;$$

2) of the kind

$$“\alpha_1, \dots, \alpha_n (1 \rightarrow n) \beta”,$$

where $\alpha_1, \dots, \alpha_n, \beta$ are phrases and $1, 2, 3, \dots, n$ are indexes of phrases, means that the phrase β is a part of the whole compound phrase which parts are also the phrases $\alpha_1, \dots, \alpha_n$; or, otherwise expressing the same, as a result of the use in the given case of the operator $“(\rightarrow i)”$ we obtain the phrase

$$“(\alpha_1, \dots, \alpha_n) \beta”,$$

so, consequently not the phrase

$$“\alpha_n \beta”$$

not a part of the phrase numbered by $“\alpha_n”$.

Note 2 to section 0.1.1. Using an explication of the concept “task” as well as taking into account the analytical way of general formulation of the task of present investigation (in accordance with the requirement formulated in this section 0.1.1, item “4”),(1)), we can state:

1) the items “1)” and “2)” taken together offer a formulation of *the order* of task of present investigation, consisting in (1) *the demand* (or: *immediate order*) of the task, or, more precisely speaking, the demand in the order of the task, i.e. what is demanded in the order of the task to be done in order the obtained result to be accepted as its accomplishment (solution) – the item “1)” mentioned above, and (2) *the remote purpose* to which are subordinated (or, otherwise speaking: are intended) the results which have to be obtained as a solution of the task – the item “2)” mentioned above;

2) the items “3)” and “4)” taken together offer description of *the conditions* of solution of the task, consisting in (1) *presuppositions*, expressing *what is given* (or: *what is taken as given*) in the posing the task – the item “3)” mentioned above, and (2) *the requirements*, expressing what is required in order the proposed solution of the given task to be well substantiated.

Note 3 to section 0.1.1. It is useful to formulate the following analogy of the structure of a task and the process of its solution, on the one hand, with the state of a material point and its movement in accordance with the way of their description in the classical mechanics, on

the other hand: **(1)** the conditions of solution of the given task (involved in the formulation of the task – cf. note 2 above, item “2”) correspond to the co-ordinates of the material point, expressing its passive state, and **(2)** the order of the task corresponds to the impulse of the material point and expresses the dynamical factor of the task giving an impetus to it for making the next step of the process of its accomplishment and in the same time determining the direction of this “next step”. Besides, the cases when the realization of a “next step” just mentioned results into addition of new elements in the conditions for solution of the task as well as into more concrete order (respectively, suborder) for making a new “new step” with new direction, correspond to change of the state of material point, i.e. to change of its co-ordinates and impulse as result of realization of its elementary movement (determined by its previous state). So, after all, we obtain *a mechanical model of the process of accomplishment of a task as movement of a material point changing the value of its “co-ordinates” and “impulse” in the logic-semantic space.*

Note 4 to section 0.1.1. The “preliminary, very general... view about what is (1) the science, and (2) the philosophical problems of science”, mentioned in the item “3)” of the basic text of section 0.1.1, characterized explicitly as “*intuitive*”, can be characterized also as *partially empirical*. The last one follows from the explicitly expressed fact that this view is “formed on the basis of experience obtained during my personal occupation until now with study in the field of some sciences as well as with treatment of their philosophical problems”. Indeed the concept “experience” contains an empirical component which in the given case consists in dynamic interconnection of my personal mind, respectively of my personal thinking, on the one hand, with the real facts of scientific knowledge, treated as something intersubjective, i.e. as something outside of my personal mind and of my personal thinking (besides the last mentioned is not the same as objectivity of the matter and of the material phenomena in the epistemological sense of the word, i.e. in respect to the humanity as a total subject of the thinking and knowledge), on the other hand.

Note 5 to section 0.1.1. Here I shall make an explanatory comment concerning the title of the section “0.1.1”: “General Formulation of the Task”, and then I shall draw conclusions from it for the direction of the first step of present investigation. In particular I have to explain here why the formulation of task in the very outset of the text of present monograph is characterised and named as “general”.

Formulating the basic text of the first (starting) section of present monograph “0.1.1” I accept the following about it:

1) First of all we have to pay attention to the fact that formulation of the order of task of the present investigation in items “1)” and “2)” is *insufficiently definite* (respectively, it is *essentially indefinite*) in respect of immediately starting the proper process of its implementation. Therefore, in order to begin the proper process of implementation of the order of given task it is necessary preliminary to formulate it in a more exact form.

2) On the second place we have to take into account that the required above more exact formulation of the order of given task in principle can be understood not as something like dusting a table, i.e. not merely as pushing aside something superfluous, however, taking into account Rudolf Carnap’s semantic conception, as a choice of one element from a set (containing more than one element) of possible explications (i.e. roughly speaking: of possible more definite modifications) of its essentially indefinite content, in the capacity of

most (or, more precisely: of almost most) appropriate explication of it, opening the possibility to immediately start the proper process of accomplishment (respectively, of solution) of the given task.

3) Posing the task generally formulated in present section 0.1.1 involves the presupposition that the description of conditions for its accomplishment in the items “3)” and “4)” is clear at least to such a degree which is necessary for initiation of the process of accomplishment of the given task (besides it is not excluded to be needed in any case to formulate some terminological explanations). Indeed, if we do not accept such a presupposition, then it would turn out to be impossible to start the process of accomplishment of the given task.

It is useful to be mentioned that the three times using the adjective “analytical” in formulation of requirements of the task in section 0.1.1, item “4)”, shows that at least in some places the form of exposition of the present investigation has to be essentially different from the narrative form of expression in a novel and nearer to the specific form of expression in mathematical and natural scientific texts.

4) It is quite possible further to realise additional elucidation and enrichment of the content of concepts involved in the formulation of the conditions for the accomplishment of the given task as well as of its order and so, step by step to obtain a chain of accepted explications expressing more and more precise and detailed formulations of the task, initially formulated in the main part of the present section 0.1.1, which can be treated as a choice from a set (containing more than one element) of possible chains of explications of the same shape. So, it turns out that with respect to the formulation of task in section 0.1.1 a set of successive explications of it generating a chain as well as a more powerful set of various possible chains of its explications corresponds; otherwise speaking this means that the formulation of task in section 0.1.1 expresses a *general idea* which is liable to be concretised in different ways by means of its different possible explications – namely this state of affairs is taken into account in the case of using the term “General Formulation of the Task”.

0.1.2. General formulation of the specification of task of present prolegomena

Taking into account that the implementation of the task generally formulated in section 0.1.1 (taken together with the subsequent notes 1-6) is too complicated owing to the very strong scientific requirements of (1) exact formulation of the essential propositions in an analytical way and (2) logical and empirical argumentation of the accepted propositions, also in an analytical way (besides there is connection between fulfilment of the both types of requirements), the author of present text considers as rational the following way of action: parallel to the work directed to construction of an interdisciplinary approach to foundations of science fulfilling the very strong requirements mentioned above to prepare also an *abridged and simplified* presentation of it as prolegomena to the adequate exposition of the given topic.

Note to section 0.1.2. Indeed, a principle disadvantage of such kind of prolegomena would turned out that the treatment of the topic in it would be not at a high scientific level (with all negative consequences of the last one), however this disadvantage is connected and compensated with the advantage that such kind of abridged and simplified presentation of the given investigation would allow the reader easier and faster to make acquaintance with the basic results of the present investigation.

0.1.ω. Corollary from sections 0.1.1 and 0.1.2

Taking into account what was mentioned in the note 5 to section 0.1.1, in order to make transition from *the general formulation* of the task of investigation in section 0.1.1 to its *solution* it is necessary to make *a previous explication* of the formulation of order in it at least in such a degree which is sufficient for such a purpose, by means of its semantic analysis, using the description of *conditions* of its solution formulated in section 0.1.1, items “3)” and “4)” as well as taking into account the requirements of abridgement and simplification mentioned in section 0.1.2.

0.2. Semantic analysis of the order in general formulation of the task of elaboration of an interdisciplinary approach to foundations of science

0.2.0. About the way of explication of the order in general formulation of the task of elaboration of an interdisciplinary approach to foundations of science

Thesis 1. In order to make appropriate explication of the order in general formulation of the task of present investigation (according to the corollary 0.1.ω) it is necessary previously to make explication of the content of basic concepts (respectively, of the sense of key terms) in it by means of their semantic analysis step by step directed from the simpler to more complicated concepts (respectively, terms) in the following order:

- 1) the concept “science”,
- 2) the concept “foundations of science”
- 3) the concept “interdisciplinary foundations of science”.

Thesis 2. In order the explication of the three basic concepts (of the general formulation of the task of present investigation) mentioned above in thesis 1 to satisfy the requirement of empirical substantiation, their semantic analysis has to rely on the history of science (besides the last one requires for theoretical elaboration of the empirical data to be used inductive logical reasoning).

Thesis 3. Taking into account the facts that **(1)** the science as well as the concept of it are formed to some degree and form in the course of historical

process of development of science, so that the author of present text has taken this concept from the conceptual arsenal of history of science, on the one hand, however (2) the concepts “foundations of science” and “interdisciplinary foundations of science” in their general form are essentially introduced here, indeed taking into account some presuppositions of them in the history of science (1→2) we have to accept following two propositions:

1) To propose explication of the general concept (1) *of science* means to put to the intuitive, essentially indefinite general concept of science which takes place in the scientific and philosophical discourse into correspondence an explicitly well defined (with respect to present investigation) general concept of science; (2) *of foundations of science* and *of interdisciplinary foundations of science* means to construct explicitly well defined (with respect to present investigation) general concepts of them, besides the well defined general concept of foundations of science has to embrace in essence as partial cases the specified concepts which take place in the scientific and philosophical discourse as “foundations of mathematics” and the like, or at least to conform in some degree and form to them..

2) The question about empirical substantiation of their explication on the basis of history of science has to be raised here in correspondingly two different forms, namely: to reveal in the history of science the processes of formation (1*) of the characteristic features of sciences, and (2*) of the presuppositions of introducing the general concepts (2.1*) of foundations of science and (2.2*) of interdisciplinary foundations of science.

Thesis 4. In order the empirical and inductive logical substantiation of the explication of the three basic concepts mentioned in thesis 1 to satisfy the requirement of *analytical way* of its implementation it is necessary to fractionalize the continual process of historical development of science transmuting it in an appropriate way into a set of separate facts of the history of science.

Thesis 5. The concrete character of fractionalization of the continual process of historical development of science mentioned above in thesis 4 has to satisfy here following two requirements for rational restrictions:

1) According to the general methodological principle of economy of thinking this fractionalization has to be accomplished only in some extent and in such a form which is corresponding to the need of a given investigation; so, in the present case we have to include in the set of historical facts of development of science only those events and in such a form which are relevant to the

problem of explication of the concepts “science”, “foundations of science” and “interdisciplinary foundations of science”.

2) In order the set of separate facts of the history of science mentioned above in thesis 4 to satisfy the requirements of the present prolegomena concerning abridgement and simplification of the exposition of given topic (cf. section 0.1.2) it turns out to be rational to reduce here the scope of events of the history of science which have to be included in the set of separate facts of the history of science relevant to the problem of explication of the concepts “science”, “foundations of science” and “interdisciplinary foundations of science” to the most important in this respect facts of the history of (1) mathematics, (2) physics, (3) chemistry, (4) biology and (5) philosophical problems of science.

Corollary from theses 1-4. In accordance with theses 1-5 we have

1) first of all to construct a set of basic facts of the historical process of formation (1) of the characteristic features of science, and (2) of presuppositions for generating the general concepts (2.1) of foundations of science and (2.2) of interdisciplinary foundations of science (1→2) which are relevant to the problem of explication of the concepts “science”, “foundations of science” and “interdisciplinary foundations of science”;

2) on the second place (1) by means of carrying out analysis of facts of historical development of science included in the set mentioned above in item “1)” and (2) taking into account some considerations of rational character (1→2) successively to propose appropriate (with respect to the task of present investigation) explications of the concepts “science”, “foundations of science” and “interdisciplinary foundations of science”.

0.2.1. The set of basic facts of the historical process of formation (1) of the characteristic features of science and (2) of presuppositions for generating the general concepts (2.1) of foundations of science and (2.2) of interdisciplinary foundations of science

Preliminary note to section 0.2.1. The text of present section 0.2.1 is composed by means of using a great amount of sources in the field of history of science among which it is worthy to be especially mentioned [1]-[8].

Fact 1.

The process of origination of first branches of science and of European philosophical tradition began in ancient Greece during 6th c. B.C. to a considerable extent as a result of rational reconsideration (1) of the

achievements of pre-scientific knowledge of the more ancient eastern peoples obtained in their practical activity, as well as **(2)** of their mythology which also has its root in their experience. .

Note. Consequently, the establishment of facts of formation of the first elements of characteristic features of the sciences in ancient Greece presupposes a short account of the characteristic features of the pre-scientific knowledge of the more ancient eastern peoples.

Fact 2.

On the basis of literature about the history of science in the remote past we can formulate the following eight characteristic features of the pre-scientific knowledge of the more ancient eastern peoples in respect to ancient Greeks:

1) The pre-scientific knowledge differs from the usual so called every-day knowledge (in the strongest sense) of the people in particular **(1)** with a general social significance and in the same time with a constant significance for the actions of individual of a given type, **(2)** with its fixing in some way (for example by means of memory or written texts), **(3)** with presence of its elementary organisation realised by means of grouping of elementary units of knowledge as a relative whole, which in the large scale correspond roughly speaking to the contemporary classification of the scientific knowledge into mathematical, physical, astronomical, meteorological, chemical, biological, and the like, and **(4)** with its passing from one generation to the next generation.

2) We can make distinction between two basic forms of the pre-scientific knowledge: **(1)** immediately connected with the practical activity of the people pre-scientific knowledge and **(2)** mythology.

3) The origination of various separate units of knowledge of the both basic forms of ancient pre-scientific knowledge mentioned above (in the item “2)”) owes to two chief factors: **(1)** it is immediately based on the natural observations during the life and especially on the experience of the people in the course of their practical activity, and **(2)** it is immediately motivated by the requirements of the life and especially by the practical activity of the people. Besides the origination of various myths in particular owes to two additional factors: **(3)** it is based also on the natural great power of imagination of the ancient people, and **(4)** it is motivated by their mental requirements to adjust the great variety of life-important facts as well as to find general purport in it.

4) The pre-scientific knowledge of the old eastern (in respect to Greece) peoples is characterised by means of a great degree of permanence, stiffness, corresponding to the permanence, stiffness of the way of life under the

conditions of great despotic monarchies, where the way of life of the individual is considerably determined by the state power.

5) The conditions of great despotic monarchies, mentioned above in the item “4”, paralysing any initiative of the individual, has reflected on the way of his thinking, otherwise speaking has determined his mental adjustment: before the individual of this epoch the question was mainly *how* – how to fulfil his obligations, how to solve a problem, how to accomplish a task for producing, constructing or making something and in such a connection also the question *what* – what is this.

6) In accordance with the immediate practical motivation and the immediate empirical way of origination of the pre-scientific knowledge immediately connected with the practical activity of the people, mentioned above in the item “3)” as well as with the mental adjustment of the individuals in this epoch mentioned above in the item “5)” it is realised mainly in a form of collections of empirical descriptions of phenomena or of rules of practical activity formulated on a concrete level, sometimes merely in a form of typical examples, without substantiation and explanations. Besides some embryonic elements of theoretical consideration of the phenomena of reality involving elementary assertions of their properties, rudimentary argumentation of statements and so on sometimes take place too; (2) some explanations (however not adequate from a contemporary scientific point of view) of the phenomena have taken place in the mythology.

7) In some cases the carrier of some elements of pre-scientific knowledge immediately connected with the practical activity of the people and of some myths are priests, moreover in some cases the last ones also carry out pre-scientific investigations. In such a way the pre-scientific knowledge turns out sometime to be connected with religion and even to be considered sacramental.

8) Indicating in item “4” the great degree of permanence, stiffness of the culture, more especially of the pre-scientific knowledge, of old eastern populations we have to add now that in the same time nevertheless a slow process of progress of knowledge has taken place. Among the factors giving an impetus to development of knowledge in the remote past we can especially mention the various forms of production of material values, the navigation and the commerce (the last one requires improvement of the way of measurement and calculation as well as of other kinds of knowledge and skills, on the one hand, and promotes the dissemination of knowledge from one region to other one). As a result of accumulation step by step of small units of new knowledge during many centuries the eastern peoples (in respect to ancient Greece) have

reached considerable achievements of pre-scientific knowledge during 6th – 3rd c. B.C. The state of pre-scientific knowledge is different for different politico-geographical regions. So, during that epoch especially permanent was the pre-scientific knowledge in Egypt and China; it was more changeable in direction of enrichment in Phoenicia – particularly in consequence of the sea-trade, and in Babylon – particularly because various trade ways were crossing there. In this connection we can mention on more concrete level the following:

8.1) The archaeological collections of implements of production, well or partially preserved constructions of various kinds and other kinds of historical evidences show that old eastern peoples possessed during this period **(1)** considerable knowledge about the physical and chemical properties of various forms of the matter as well as **(2)** good ability and skills to use this knowledge for their practical aims.

8.2) The historical evidences concerning the medicine and agriculture of eastern peoples (in respect to ancient Greece) during 6th – 3rd c. B.C. show that they possessed important knowledge about the life as well as knowledge of meteorological character - about change of day and night, change of seasons and so on.

8.3) The observation of sky during every-day life of the peoples resulted statements of various astronomical facts, particularly it is worthy to mention the attainments in the pre-scientific astronomy of the Egyptians, Babylonians, Chaldeans and Phoenicians; the accurate study of sky by the Phoenicians was motivated in particular by the requirements of navigations.

8.4) The pre-scientific mathematics, i.e. the practical knowledge of numbers and space obtained mainly in an empirical way, attained great success especially in Egypt and Phoenicia and namely they were of great importance for origination of mathematics in ancient Greece.

8.4.1) The pre-scientific arithmetic has been especially developed in Phoenicia, Babylon, Chaldea and Egypt. Tables of the numerical values of some arithmetical functions were made. Much attention was paid to multiplication of integers and calculations of fractions – this attention can be explained by means of their importance for the process of production, commerce as well as for the assessments and imposing taxes, on the one hand, and by means of difficulties in that time to be fulfilled. Multiplication of integers were reduced to repeated doubling of the multiplicand and subsequent appropriate addition of the results of doubling; for the purposes of operation with fractions the last ones were represented by sums of fractions the numerator of which is unity.

8.4.2) The origination of pre-scientific geometry is chiefly due to Egypt and it is especially connected with the periodical inundations of Nile resulting into sweeping away the landmarks. Egyptians were able to obtain right angle in the process of construction of buildings using the so called Pythagorean theorem for a particular case, namely constructing a triangle the lengths of which sides are 3, 4, 5. They have attained a good approximation of the constant $\pi = 3,160\dots$

8.4.3) In accordance with some evidences the pre-scientific mathematical knowledge in old China can be traced back to the 14th c. B.C. and they were increasingly developed during the next centuries.

Fact 3.

1) The transition from the pre-scientific knowledge in Asia and North-East part of Africa (Egypt) to the scientific knowledge in Europe is due to a significant event in the history of humanity: earlier than thousand years B.C. four Greek tribes – Aeolians, Dorians, Achaeans, and Ionians – came from somewhere of the Nord into south-east part of Europe, or more precisely of the Balkan peninsula, initially settled there and bit by bit they made settlement of the islands in the Mediterranean as well as of its east shore (i.e. in the western shore of the Asia minor). The Greeks were not merely a new nation (under the last word we have in mind the amalgamation of the four tribes mentioned above); It is much more important that they had another social being, another way of life in respect to the people in the eastern countries and correspondingly the Greeks as individuals possessed another mental adjustment, another spirit disposition in respect to the individuals in the eastern countries.

2) Settled in the territory mentioned above the Greeks erected their cities as commercial centres. Contrary to the great old despotic eastern monarchies the Greek cities together with their vicinity formed small states – the polities. Under the conditions of such a kind of state organisation the Greek citizens felt themselves as free people, i.e. as individual who may take proper decision for his proper action. The government in some polities was aristocratic, in the rest polities it was democratic. In the last case the businessmen had to do not only their overseas trade, not only to buy some goods in one place, to travel and afterwards to sell them in another place, but they had also to play a role in taking governmental decisions about the life in their polity. Such social conditions of life stimulated them to maintain their dignity, to enlarge their cultural interests, to strive to widen their outlook, to improve their perception of the encircling world, in particular to make explication of the observed phenomena which presupposed to put not only the questions *what* and *how*, but also the question *why*. All these means, otherwise speaking, the new social

conditions of life in Greece stimulated the rational way of thinking, including as an essential element the striving for explanation and argumentation. Besides the hard work of slaves provided them with leisure time for intellectual activity.

3) So, the Greek merchants, in the course of executing their business visiting eastern countries and coming into contact with their old civilization, perceived it in their manner, rationalizing its achievements, besides in the same time it (i.e. the old eastern civilization) stimulated and directed their creative, innovatory thinking and in such a way a process of origination of science and philosophy (in its European tradition) initiated. Raising the question “why?” in case of treatment of natural or social phenomena it obtained actually the meaning of a question about the cause of the observed phenomena and in such a way the first steps of process of origination of natural sciences and humanities started; raising the question “why?” in case of treatment of numbers and space on an abstract level, influenced by the great achievements of their empirical treatment closely connected with concrete practical questions in the Eastern countries, it obtained actually the meaning of a question about the logical reasons of acceptance of these or other statements about them and in such a way the first steps of process of origination of mathematics started; raising the question “why?” in case of treatment of the very general and fundamental problems of the being and of its knowledge it obtained actually the both meanings - of a question about the cause of the observed phenomena as well as of a question about the logical reasons of acceptance these or other statements about them and in such a way the first steps of process of origination of philosophy (in its European tradition) started.

4) The very first steps of the process of origination of science, humanities and philosophy obtained a positive social resonance – they were accepted by the Greek society as social value. As a result many young men devoted themselves to philosophy and science – at the beginning to learn it by the masters and then to develop and to teach them. In the same time the question “why?” of both meanings - as a question about the cause of the observed phenomena as well as as a question about the logical reasons of acceptance these or other statements supported the spirit of critical attitude to various assertions living. As a result of such social conditions various philosophical and scientific schools flowered in the ancient Greece and entered into serious discussions between them, which stimulated a dynamical development of the philosophical and scientific thought in a great degree.

Note to fact 3. The content of the present fact 3 will obtain more concrete form by means of the exposition of the subsequent facts 4.

Fact 4.

The founder of the philosophy (in its European tradition), mathematics and astronomy (more generally speaking: natural science) in the ancient Greece was Thales from Miletus (about 640 – about 550). He was a businessman and in this connection he visited Egypt where during his leisure he made a study of geometry and astronomy. In his native city Miletus he was engaged in public affairs. In accordance with some legends he applied his knowledge in his business resulting great success. He was honoured as one of the seven sages of Greece. He is considered **(1)** as the founder of philosophy (in the sense of European tradition) especially with his statement that the water is the first principle of all things, **(2)** as the founder of mathematics - on account of proposing for the first time proofs of six geometric theorems about angles, determination of triangles, circles (at least for some of them in a very primitive form) and **(3)** as the founder of astronomy in Greece (only a few his statements about the earth as a celestial body are known at present – about duration of the year on the earth and that the last one is disk-like celestial body floating on water), besides in such a way it turns out that he is also more generally speaking the founder of Greek natural sciences. In addition he is the founder and the first head of Ionian school in philosophy, mathematics and astronomy which had proceeded and further developed his intellectual work.

Fact 5.

Thales's pupil Anaximander (611 B.C. – 545 B.C.), as it seems in accordance with the existent historical data, developed further the intellectual work of his master in the all three fields mentioned above (in the content of fact 4) elaborating theories in those fields in which Thales had enunciated only separate statements (let us remember: in mathematics these statements are not merely sentences, however theorems supplied with proofs) and raising the level of treatment of corresponding subjects: **(1)** for the field of philosophy it is worthy to mention especially that he enunciated the first principle of the things to be *ἀπειρον* – a substance which is something more abstract than the water, besides the various other things are generated by its condensation and rarefaction (the first historically known attempt to propose mechanical explanation of the variety in the forms of material things); **(2)** for the field of mathematics it is known that he wrote a book on geometry which, naturally, is a next step of its formation as a separate scientific doctrine; **(3)** in astronomy he created a model of the cosmos expressing a geometric structure of it. Also various scientific inventions are attributed to him as sun-dial, the first Greek geographical map, terrestrial and celestial globes.

Indeed the scarce information about the achievements of Thales and Anaximander do not allow us to make a categorical and exact comparison of their level and value, however nevertheless I am inclined on the basis of the available information to assert that the achievements of Anaximander in these three fields of the knowledge had overtopped the ones of Thales and namely in such a way the initiative work of Thales in the history of science and philosophy was categorically proceeded and obtained consolidation as a necessary condition for its further development.

Fact 6.

A great step forward in the historical process of formation of mathematics as a science had made Pythagoras (about 570 B.C. – about 490 B.C.) who was born in Ionic island Samos and initiated another philosophical and mathematical school in Italy. As a native of Ionic island, naturally, at first he was influenced by the physical philosophy and mathematical investigations of Thales and Anaximander. At the suggestion of Anaximander he went to Egypt, spent there some years, afterwards he travelled in Asia Minor, and after some events in Greece he settled in south Italian city Croton where founded a successful school in which had developed his doctrine – philosophical, mathematical, religious, political (one may say the Pythagorean school was in the same time correspondingly philosophical, mathematical, religious, political, too). Here we shall mention only following six items of the Pythagorean doctrine (under the last term we shall have in mind not solely the ideas of Pythagoras only, however the ideas developed by the whole of members of the Pythagorean school) which are relevant to the process of formation of mathematics as science and of the philosophy of science: **(1)** the geometry was further developed in the Pythagorean school after Thales and Anaximander enriching considerably its content as well as giving it a more rigorous deductive character, **(2)** Pythagoras introduced a new subject of mathematical investigations in Greece – numbers, more precisely speaking the rational numbers identifying the concept of number with the contemporary concept of rational number; **(3)** in distinction from the way of treatment of numbers in old East civilizations exceptionally as elements of calculations their treatment in the Pythagorean school is on a more abstract level directing the attention chiefly to their forms and properties, so that using a modern terminology we may speak about arithmetic as well as about *theory of numbers* in the Pythagorean school; **(4)** the distinction in the treatment of numbers mentioned in the previous item (3) is determined by the distinction in the reason for the intellectual work on them – the reason for study of number calculations in old East civilizations consisted in

the view of their practical importance, mainly for the processes connected with producing of goods, imposing taxes and commerce, while the reason for study of forms and properties of numbers in Pythagorean school consists in the view of their *pure cognitive significance* in accordance with their philosophical treatment as principle of all things, otherwise speaking with treatment of things as imitation of the numbers, pushed into the foreground as generalization of the established empirical facts that the harmonious chord of three strings is determined by a definite relation between three numbers expressing their length as well as that such a kind of harmonious relationship takes place for the radiuses of heavenly bodies, anticipating in such a way the contemporary conceptions of mathematical modelling of natural and social entities, processes, systems; (5) the discovery of incommensurable geometrical magnitudes (from the point of view of the system of rational numbers) which is in contradiction with Pythagorean statement about the numbers as the essence, respectively as the grounds, of all things, resulting in such a way the first crisis in mathematics; (6) the religious character of the Pythagorean school giving the whole Pythagorean doctrine and in particular the mathematical part of it also a spiritual character – to be obligatory and in the same time secret for the members of the school, so, in such a sense to be closed for the school.

Fact 7.

The closed character of mathematical knowledge in Pythagorean school (in the sense mentioned in fact 6, item 6) probably was very useful for its intensive development from the very first days of its existence, whoever later on the secrecy of mathematical knowledge obtained in the school turned out to be entirely artificial for those its members who have devoted themselves just to mathematical research and moreover it turned out to be even a hindrance for its further development. After the death of Pythagoras, these members of the school separated formally from it, broke the requirements of secrecy of the doctrine, in particular of the mathematical knowledge as its part, and proceeding with their intensive work in the field of mathematics in the spirit of Pythagorean tradition they disseminated the obtained results. So, as a result of this act the mathematics obtained autonomy as a science and at the same time a new type of intellectuals appeared – mathematicians. This was an essential step towards formation of mathematics as a separate, self-dependent science; one may say it was completion of the first stage of the historical process of formation of mathematics as a science. Besides, ending the first stage of the historical process of formation of mathematics as a science was naturally followed up by the opening the next, second stage of it.

Fact 8.

The development of the philosophy from the 6th c. B.C. till the beginning of 4th c. B.C. (from Anaximenes of Miletus till Democritus of Abdera) characterized by underlying of different, sometimes opposite features of the reality and of its knowledge by the different philosophers and philosophical schools provoking deep discussions. Much of these philosophical conceptions, approaches and discussions may be regarded as embryonic elements of the future philosophy of science, developed parallel with the process of development of embryonic elements of the language, problems, approaches and content of future sciences in their framework.

At the end of this period of time Democritus (about 470 B.C. or 460 B.C. – about 370 B.C.) made essential contribution to the process of formation of philosophy of science **(1)** first of all by means of combination of empirical studies of nature (there are evidences that he has even dissected animals in order to study them) with application of high level of analytical rational thinking connected with the endeavour to penetrate in the internal essence of the things, beyond their external visible aspect; and **(2)** on the second place, as a result of application of the approach to study of nature expressed in the combination of empirical and rational forms of research, mentioned above in the item (2), by means of further elaboration of the Greek atomism initiated by Leucippus (5th c. B.C.) in which one can reveal the following three aspects: **(2.1)** ontological aspect – in reality there are only atoms and the void, so, all things of the nature, including the traditional in ancient Greek philosophy elements earth, air, water and fire, as well as the soul, are some kinds of clusters of atoms which can be of different type, besides namely the properties of the atoms of which they are consisted determine their properties; **(2.2)** epistemological aspects – the various things of the nature don't exist in that forms in which we perceive them by the five senses, however in truth they are merely some kinds of collections of great number of invisible atoms; and **(2.3)** methodological – both aspects just mentioned, the ontological and the epistemological, can be regarded as manifestations of two approaches to research the nature respectively - **(2.3.1)** analytical-synthetic approach to explanation of the natural phenomena consisting in the requirement on the first place of revealing the last elements of their substance which can turn out to be invisible and further by means of the properties of last ones to explain the visible properties and behaviour of the studied phenomena, and **(2.3.2)** the information immediately obtained by means of using empirical way of study of the natural phenomena requires a critical attitude and revision by means of application of abstract analytic thinking.

Fact 9.

A decisive step towards formation **(1)** of the philosophy of science and as its realization **(2)** of the mathematics as a science, **(3)** as well as in some degree also of the astronomy as a science **(1→3)** made Plato (about 428 B.C. – about 348 B.C.) and his Academy. An interchange of the influences between **(1*)** the content of Plato's philosophy of science (at the course of its formation) and **(2*)** the form of mathematical thought of the Pythagorean mathematicians (at first before foundation of the Academy and later in the Academy) **(1→2)** had taken place, hence the independent, separate exposition below of the items **(1*)** and **(2*)** just mentioned is conditional.

1) One may distinguish in *Plato's philosophy of science* following three parts: on the first place **(1)** the ontology; on the second place, built on the basis of the ontology - **(2)** the epistemology; and on the third place, built on the basis of the ontology and the epistemology taken together - **(3)** the methodology.

1.1) Plato's ontology of science. The starting point of view of the Plato's ontology is his thesis about distinction between two worlds: **(1)** material – the world of material things, which are changeable, uncertain, in the spirit of the Heraclitus, and **(2)** ideal – the world of the ideas which are eternal and unchangeable, in the spirit of Parmenides, or, using a contemporary terminology one may say: the world of abstract mathematical structures as ingenerating models. There is the following relations between these two worlds: the basic, fundamental is the world of the ideas and the material world is secondary: the material things are reflections (i.e. ingenerated models) of the corresponding ideas.

1.2) Plato's epistemology of science. Correspondingly to the distinction between two worlds and the treatment of their relationship in the Plato's ontology, mentioned above, a basic role in Plato's epistemology plays the distinction between following two forms (and in the same time: following two grades) of knowledge (using a contemporary terminology): **(1)** of a lower grade - *empirical knowledge*, which **(1.1)** is obtained by means of sensations and **(1.2)** is giving information about material things, so the class of last ones is called respectively “a visible region of objects”; and **(2)** of higher grade - *rational knowledge*, which **(2.1)** is obtained by means of thinking and **(2.2)** is giving information about the ideas, so the class of last ones is called respectively “an intellectually comprehensible class of objects”.

Each of these two basic forms of knowledge is further divided into two subforms, representing two its subgrades. So, **(1*)** the empirical knowledge is divided into the following two subforms: **(1*.1) conjecture** – the meter of

opinions represented by the images of visible objects as their shadows, reflections in water and so on, and **(1*.2) belief** - the meter of knowledge obtained by sensation of the real objects (corresponding to the images mentioned above); **(2*)** the rational knowledge is divided into the following two subforms: **(2*.1) judgement** (in German: *das Urteil*) – the way of thinking starting in a given discourse from a set of preliminary accepted presuppositions as definitions and so on, besides possibly using auxiliary drawings (however using them the thought is busy not with the figures drawn in it, but with the originals reflected into the drawings), step by step the thought is moving to the conclusion about the subject of the discourse, and **(2*.2) reason** – the way of thinking is directed from the presuppositions, preliminary accepted in a given discourse, to the first principles which are not presupposed by anything (so, in such a way the basis of the accepted presuppositions mentioned above in the item (2*.1) is revealed, otherwise speaking the last ones obtain their substantiation and explanation).

1.3) Plato's methodology of science. It is quite possible that at the first glance the ontological and epistemological conceptions of Plato, concisely formulated above, look very abstract and far from the problems of the development of science. However they played a role of basis for formulation of methodological judgements about scientific knowledge, which turned out to be very original and in the same time very fruitful for the further development of the science – therefore they were named above “ontology of science” and “epistemology of science”.

According to Plato there are the following correspondences to the forms of knowledge mentioned in item “1.2)” and to two worlds mentioned in the item “1.1)”:

1.3.1) The sensation, giving information about “a visible region of objects”, respectively the empirical knowledge (if we prefer to use a contemporary term), is not able to give a reliable basis for elaboration of sciences and philosophy - actually to it corresponds the every-day knowledge and it is hardly possible Plato to have denied its role in the every-day life of the people, but such a circumstance is decisively out of the problems of his aristocratic philosophy).

1.3.2) To the place of judgement (cf. item “1.2), (2*1)”), directed to obtain comprehension of “an intellectually comprehensible class of objects”, namely of the ideas, Plato put the sciences: mathematics (arithmetic and geometry), astronomy and music. This localisation of the sciences among the forms of knowledge determines the following methodological requirements to them: **(1)** for the mathematics – to formulate preliminary the presuppositions of the mathematical discourse, which in contemporary terminology means to

formulate preliminary its axiomatic basis, and **(2)** for the astronomy and music - the study in these two fields of science has not to be directed to that which is visible or hearable, respectively, but it has to be carried out like in geometry – by means of application of general propositions, revealing qualitative relations and real forms, which in contemporary terminology means that it (the study in these two fields of science) has to take the form of mathematical modelling.

1.3.3) To the place of reason directed to research of the presuppositions (otherwise speaking – of the foundations) of sciences (cf. 2*.2) Plato put the dialectic, i.e. using a contemporary terminology - the philosophy of sciences.

1.ω. This Plato's philosophy of science, sketched out above and expounded by its author by means of artistic skill, interlaced with consideration of such kind of philosophical problems which at present are referred to philosophy of life, turned out to be very fruitful for the historical development of philosophy and science, but not as a dogma, however as a starting point of further specifications at the course of the history of philosophy and science. Even his famous contemporaries and pupils Eudoxus and Aristotle, who worked hard at the questions about the nature based in some degree on empirical observations, had criticised Plato's sharp discrepancy between the matter and ideas, the single and the general, the empirical knowledge (i.e. the knowledge of "visible region of objects") and the rational knowledge (i.e. the knowledge of the "intellectually comprehensible class of objects"), making endeavour in their philosophical reasoning to overcome this discrepancy. One can say that the basic distinctions in Plato's ontology, epistemology and methodology of science mentioned above are in essence preserved, naturally with important modifications corresponding to the contemporary state of affairs of the contemporary science.

2) The intensive development of mathematical research in Pythagorean school connected with construction of proofs of mathematical propositions consisting in successive, step by step drawing some mathematical propositions from other ones results into revealing a definite logical order among them. Namely revealing such a kind of logical connection between mathematical propositions expressing application of the axiomatic method as the instrument for construction of mathematical theories and pointing out at the ideal character of mathematical objects and of the relations between them influenced Plato to formulate his philosophy of mathematics mentioned above (cf. item "1.3.2), (1)"). The last one for its part stimulated essentially the mathematicians in the Academy to elaborate in an axiomatic way some mathematical theories which subsequently entered in the Euclid's Elements.

Fact 10.

(1) Archytas according to Diogenes Laertius was the first who had put in order the problems and statements of mechanics laying down of them mathematical foundations as well as he was the first who had reduced the movement of mechanisms to geometrical drawings; (2) Archytas and Eudoxus had used mechanical models to research of geometrical problems; (3) Eudoxus had constructed a mechanical celestial model consisting of 27 spherical disks to which various celestial bodies were attached and by means of their rotations various observed effects were obtained. All these inventions carried out an essential step towards formation of mechanics and astronomy as sciences.

Fact 11.

The next, decisive step in the process of formation of the natural sciences as well as of the philosophy of science is realized by Aristotle (cf. [9]). In addition to that, what was mentioned above in the item 1.ω about Aristotle's position in philosophy (which is relevant to the philosophy of science) in its relation to the Plato's one, we shall underline below following three his principle contributions to formation of the philosophy of science:

1) First of all Aristotle, together with the Megaric school, is pioneer of laying down historical foundations of logic however in two fundamentally different forms, which especially after its historical development turned out to be of basic importance for construction of the contemporary philosophy of science: the Megaric logic was a predecessor of the contemporary classic propositional calculus; the Aristotle's syllogistic was a predecessor of the contemporary classic predicate logic of first degree. For the positive information the process of arrival at a conclusion from premises in the syllogisms roughly speaking is directed from the general cases, i.e. from such kind of classes of objects for which the given property or distinguishing feature is inherent in every element of them, to the general or particular case, i.e. for such kind of class of objects for which the given property or distinguishing feature is inherent in every element or in only some elements of it.

2) On the second place we shall mention that (1) in accordance with the role of the terms in the syllogisms Aristotle had paid attention to the structure of concepts, to the relations between them and to the precision of their sense in the philosophical discourse, so, in such a way he had elaborated implicitly elements of logical semantics as well as had demonstrated its significance for philosophical discourse, (2) besides obviously taking into consideration the role of the general propositions and simultaneously of the general concepts in the

syllogisms he had initiated the elaboration (in the framework of the logical semantics) of the theory of categories. Both what was mentioned just above in the items (1) and (2) is of principle importance for the philosophy of science and can be treated as laying down initial elements of its formation.

3) On the third place it is worthy to be mentioned that Aristotle had systematically treated several topics in the field of the study of nature as physics, meteorology, biology, psychology, which historically plaid an essential role in the further development of scientific knowledge. Besides it is very important to take into consideration that the Aristotle's works in the field of natural-scientific topics is not equally for different topics, respectively for different sciences in the contemporary sense. So, **(1)** his theories on the problems of physics were not based on adequate methods of research in this field of the knowledge – indeed Aristotle was not acquainted with the experiment – and as a result they turned out to play a historical role of *bridge-head to initiation of construction* of the corresponding scientific physical theories provoking the scientists during 16th and 17th centuries to show their erroneoususness by means of application of adequate experimental methods; **(2)** his investigations in the field of biology, on the contrary, can be estimated as *the first step of formation* of biology as a science taking into consideration that his description of considerable amount of various animal species was based on application of two really scientific methods of research in biology: **(2.1)** he had used dissection of the bodies of animals and, which is a great his merit in the history of biology, **(2.2)** he had introduced and first used the method of classification of the described biological units (which plaid so great role in the biological investigations carried out later, during 18th and beginning of 19th century by Linnaeus and Cuvier).

Fact 12.

1) The tendency to an increasing degree to treat the problems concerning figures and numbers in an abstract, logical way, initiated by Thales, enriched with Plato's philosophy of mathematics and Aristotle's logic and philosophy of science, brought to a conclusion in Euclid's "Elements" (cf. [10]).

2) Before Euclid's "Elements" there were in ancient Greece mathematicians, mathematical investigations, mathematical results of investigations, even separate mathematical theories appeared, but still there was no mathematics in a strong sense - as wholeness, i.e. as a theoretical whole, i.e. as unified, systematically developed science, the various elements of which are connected in a net by means of the logical relations of inference. By means of **(1)** elaboration of *the unified axiomatic foundations of the mathematics in its*

intirety, and **(2)** on such a basis carrying out a **(2.1)** *systematic* and **(2.2)** *uniform* **(2.1→2.2)** treatment of its problems **(1→2)** the process of formation of mathematics as a science, characterised by **(1*)** a specific subject of scientific research – namely the spatial figures and the numbers, and **(2*)** a special, characteristic method of scientific research – namely the logical proof of its propositions, brought to a close.

3) The axiomatic exposition of mathematics in Euclid’ “Elements” possesses the following two kinds of features of logical character:

3.1) The propositions forming the content of mathematics as it is presented in the “Elements” are partially descriptive, and partially prescriptive.

3.2) In the axiomatic system of mathematics exposed in Euclid’ “Elements” there are two kinds of postulated propositions – **(1)** “postulates”, which concern geometric objects, besides **(1.1)** the first three “postulates” are prescriptive propositions and **(1.2)** the rest two “postulates” are descriptive propositions, and **(2)** “common notions”, which concern to more general class of objects, namely to such kind of things, nevertheless of what kind is their nature, for which it is definite that **(2.1)** they can be **(2.1.1)** added one to other or **(2.1.2)** subtracted one from other, as well as **(2.2)** between which the relations **(2.2.1)** “one is equal to other”, **(2.2.2)** “one is greater than another”, **(2.2.3)** “one coincides with other”, **(2.2.4)** “one is whole in respect to other”, and **(2.2.5)** “one is part in respect to other” (for **(2.2.4)** and **(2.2.5)** it is presupposed that if one is whole in respect to other, than the other is part in respect to one, and vice versa).

4) Of course, this event – that by means of Euclid’s “Elements” the process of formation of mathematics as a science was ultimately finished - doesn’t mean an end of the further development of mathematics as a science. On the contrary, it turns out to become a basis for its further historic development, including also modification of the subject and method of research. Indeed, almost immediately after appearance of Euclid’s “Elements” a historical chain of investigations of great mathematicians – let be mentioned only the names of Archimedes (287 B.C. – 212 B.C.), Apollonius (260 B.C. – 200 B.C.), Ptolemy (2nd c.), Pappus (about the end of 3rd c.), Diophantus (probably second half of the 3rd c. – beginning of the 4th c.) – initiated, which not only **(1)** enriched the content of mathematics by means of new problems and theorems, however also **(2)** modified in some degree the form, the characteristic of mathematics as a science: **(2.1)** the proper development of mathematics was more and more active applied to the problems of mechanics and astronomy, besides in some cases in the form of mathematical models of treated natural systems and processes (Archimedes, Ptolomy), **(2.2)** the treatment of numbers and algebraic

problems was realised in an algebraic (not in geometric) form (especially Diophantus), etc.

Fact 13.

In consequence of some historical conditions the development of science (in exception of the logic) in Europe was hindered during a great period of time from the 4th c. to the middle of 15th c. In any case I can not see such considerable events in the history of science during this period of time, which would be worthy of mentioning here as facts of development of the concept of science in the course of its real proceeding.

Fact 14.

1) As a result of favourable historical conditions since the middle of 15th century begins a process of renaissance of the science (together with the parallel process of renaissance of the arts) and later it grew into a process of definitive formation and further development of *the classical science*, which came to an end somewhere at the boundary between 19th and 20th centuries (with some variations for different sciences).

2) In this connection it is worthy the following two points to be stressed:

2.1) On the one hand the classical science formed and developed during the period of time mentioned above took general shape which qualitatively differs from the shape of the knowledge in theoretical form formed and developed on previous periods of time and namely therefore its formation was realised overcoming the shortcomings of the previous theories and approaches, in a struggle against the former prejudices and dogmas, more concretely speaking chiefly against the shortcomings of the Aristotle's doctrine converted during the Middle Ages into dogmas.

2.2) On the other hand the process of formation and further development of the classical science was in some sense a continuation of the process of historical development of scientific knowledge (including already formed sciences as mathematics and elements of mechanics, as well as embryonic elements of formation of various sciences in the framework of all-embracing philosophical doctrines) during the previous epochs, chiefly during the ancient as well as the medieval centuries, besides even the shortcomings contained in the doctrines developed during these epochs, in particular even the shortcomings contained in the Aristotle's doctrine, which ought to have been overcome in the course of the formation of classical science (as it is mentioned in the previous item "2.1"), had plaid in some sense and degree a positive role

as a kind of raising essential problems for research in a form of their wrong solution.

Note to the item “2)” of the fact 14 in section 0.2.2.1.1. This statement can be treated as an example of the dialectic of continuity-discontinuity in the historical process of development of the science.

3) In the historical process of formation and of further development of the classical science the following two mutually opposite tendencies had taken place:

3.1) an integrative tendency – it had played an essential role in the process of formation of various special sciences by means of unification **(1)** of different until that time topics of investigation, as well as **(2)** of different elementary scientific theories, and

3.2) differential tendency (i.e. a tendency of differentiation) – it had realised in a process of successive (hierarchical) formation of different branches of a given field of the natural sciences possessing more and more narrow topics of research in the course of its development.

4) The formation and further development of the following three fundamental fields of *classical scientific knowledge* (embracing as a main body the classical science however not reducing to it) will be mentioned below: **(1)** the classical natural science, **(2)** the mathematics, and **(3)** the philosophic-methodological treatment of the problems about science (i.e. the origination of embryonic elements of the philosophy of science in the framework of traditional philosophy).

4.1) *The formation of natural sciences*

4.1.1) The most remarkable as well as the most important feature of the history of science during this period of time is *the formation of natural science(s)*. The decisive factor for this event was connected with the application of *empirical investigations*, i.e. of the two types of investigation of the natural phenomena on the grounds of experience, namely **(1)** *the systematic observation* and **(2)** *the experiment*. Besides the whole point is not merely in the fact of application of empirical investigations, namely of observations and experiences – the last ones in some elementary forms have taken place in the process of development of human knowledge during the all previous epochs; it consists in application of new, namely *scientific forms of empirical investigations* the basic two types of which will be conditionally named here *systematic observations* and *analytical experiments* (some of their characteristic features will be mentioned below in the framework of treatment of the fact 14).

4.1.2) We can list the following seven *fundamental fields of the classical natural science* (respectively, following seven *fundamental classical natural sciences*): **(1)** astronomy, **(2)** physics, **(3)** chemistry, **(4)** geology, **(5)** physical geography, **(6)** biology, and **(7)** psychology. There were (in some cases very essential) connections between them, especially between physics and astronomy, physics and chemistry, chemistry and biology.

For short some notes will be made below only about physics, chemistry and biology.

4.1.2.1) Among these seven fundamental fields of the classical natural science, respectively among these seven fundamental classical natural sciences, we can consider *the classical physics* to be the most developed as an exact science. Therefore we shall pay here some attention on the first place namely to this fundamental field of the classical natural science.

For the sake of brevity only the formation of a few branches of classical physics will be subjected to analysis below, using various sources concerning history of physics, among which it will be especially mentioned [11].

4.1.2.1.1) Formation of classical mechanics

4.1.2.1.1.0) Among the various branches of the classical physics we can consider *the classical mechanics* to be the first one which had concluded the process of its formation as a separate, independent, whole, exact science. Moreover, after its formation the classical mechanics used essential influence on the way of formation of the other branches of the classical physics. Therefore here we shall give some attention to the process of formation and characteristic features of the classical mechanics.

4.1.2.1.1.1) The decisive step in the process of formation of mechanics as a theoretical science of such a kind was the appearance of the work “*Philosophiae naturalis principia mathematica*” (“*Mathematical Principles of Natural Philosophy*”, 1687) by Newton in which, like “*Elements*” by Euclid, were formulated definitions of the basic concept of the language of mechanics and three fundamental (in a logical sense of the last word) laws about movement playing a role of axioms from which the rest propositions of mechanics were derived. A very important element in the foundations of the classical mechanics, as they are constructed in the Newton’s work, is the introduction of concept “force” merely as a quantity denoting the factor of change of the state of movement, which can be measured and calculated, but doesn’t offer any explanation revealing the internal mechanism causing this change.

4.1.2.1.1.2) The appearance of this classical work in the history of science was a critical unification and creative conclusion of various directions of

empirical and theoretical investigations in the field of mechanics and astronomy since “*De revolutionibus*” (1543) by Nicolaus Copernicus (1473-1543). In this connection it is worthy to be mentioned the following events of the historical process of preparation of the appearance of “*Philosophiae naturalis principia mathematica*”: **(1)** sun-centred model of the solar system elaborated by Copernicus in profound contrast to the earth-centred model of the universe elaborated by Ptolemy, **(2)** the systemic observations of the heavenly bodies using telescope by Galileo (1564-1642), **(3)** establishment of some quantitative laws of movement by Galileo on the basis of analytical experiments excluding the friction as a factor offering resistance in respect the movement under investigation and in such a way allowing the treatment (besides in a quantitative form!) of the movement in a pure form, **(4)** establishment of the three Kepler’s laws of movement of planets round the sun on the basis of systematic observations of Ticho Brache resulting into revealing of quantitative relations, and **(5)** Descartes’ **(5.1)** formulation of principles of knowledge theory and careful treating the methods of solution the problems, **(5.2)** initiation of elaboration of analytic geometry, **(5.3)** introducing the concept of quantity of movement and the principle of its conservation, and **(5.4)** elaboration of conception about the set-up of the world determined by vortices of three types of material corpuscles as a subject of profound criticism by Newton.

4.1.2.1.1.3) We have to add to the just mentioned above that the laying down of the historic foundations of classical mechanics in the Newton’s classical work is connected with elaboration and application by its author of **(1)** new philosophico-methodological approach to the given topic of research and **(2)** new mathematical method, **(1→2)**, besides for both one can show previous historical prerequisites for their formation.

4.1.2.1.1.4) The acceptance of the year of publication of Newton’s “*Philosophiae naturalis principia mathematica*” (1687) as the outset of a new stage of the history of mechanics – namely the stage of classical mechanics, does not mean that straight away of its appearance it began to play effectually the role as a theoretical basis of elaboration of its problems. As a matter of fact the publication of this classical work of Newton initiated immediately a historical phase (within the framework of the stage of classical mechanics) of confirmation of its ideas and approach to the topic in a struggle with Cartesian doctrine, mentioned above in the item “4.1.2.1.1.2), (5.4)”, which phase had been concluded about after seven decades consequently of the fact that Newton’s doctrine gained advantage over Cartesian one as instrument of study the phenomena on the earth as well as on the sky.

4.1.2.1.1.5) After confirmation of Newton's doctrine exposed in his classical work mentioned above a process of extensive development of classical mechanics **(1)** breadthways, embracing more and more natural phenomena and consequently differentiating various its theoretical branches, as well as **(2)** into the profundity of the thought, formulating more and more general and fundamental principles (namely in view of the last fact we named above the Newton's axiomatic system of mechanics "historic foundations of mechanics") **(1→2)** began.

4.1.2.1.1.6) The importance of Newton's "Philosophiae naturalis principia mathematica" overstepped the limits of mechanics. It had served in one or other aspect and degree as pattern (or, as a model) for construction of various other physical theories, so it had left its traces on the way of construction of various other branches of physics and besides they are especially appreciable and interesting on thermodynamics (cf. below the item 4.1.2.1.2) and electrodynamics (cf. below the item 4.1.2.1.3). Therefore and namely in such a sense this Newton's work can be considered as a *foundational* one in historical aspect for the classical physics at all.

4.1.2.1.1.7) Moreover, directly or indirectly (i.e. by means of influence of some, one or several, theories in the field of physics influenced on their part by Newton's work under question) it had influence over formation and development of various fields of scientific knowledge outside physics. Therefore the appearance of Newton's "Philosophiae naturalis principia mathematica" can be treated also as a revolutionary step in the history of classical science.

4.1.2.1.2) Formation of thermodynamics

4.1.2.1.2.1) The formation of thermodynamics generally speaking as a science concerning the processes connected with the change of heat of the physical bodies and its transformation into work had begun with publication of the treatise "Réflexions sur la puissance motrice du feu et sur les machines propres à développer sette puissance" (1824) (in English: "Reflections on the motive power of the fire and on the machines fitted to develop this power") by S. Carnot (1796-1832).

From the very outset of the treatise just mentioned above Carnot has given reason for the topic of his investigation underlying the great importance of steam-engines and simultaneously the lack of the general theory concerning their work. Besides Carnot has stressed that such a theory is not obtainable on the basis of mechanics, so its elaboration requires enlargement of the scope of physical laws as well as of their generalisation; otherwise speaking Carnot had

paid attention to the significant circumstance of methodological character that the elaboration of a theory of the heat-phenomena must be realised in a peculiar way, different from the way of construction of mechanics. Indeed, while the basic subject of research in the rational mechanics is the ideal object “material point”, the basic subject of research in the Carnot’s treatise (as well as later in the thermodynamics) is an ideal system representing a theoretical model of the heat-engine, named later “cycle of Carnot”. Also, while the rational mechanics is a pure phenomenological theory of the motion, the theory of heat-phenomena in Carnot’s treatise is based on using the concept of caloric as material substance of the heat possessing some visible explanatory functions concerning the process of transfer the heat from one physical body to another one.

4.1.2.1.2.2) During the middle of 19th century in consequence of a critical treatment of Carnot’s ideas in “Réflexions sur la puissance motrice du feu et sur les machines propres à développer cette puissance”, taking into account some new results of empirical investigations and eliminating the concept of caloric as unfounded, J.P. Joule, W. Thomson (Lord Kelvin) and R. Clausius had laid down the foundations of phenomenological thermodynamics, based on two principles formulated mathematically – **(1)** the principle of conservation of energy (formulated in a way appropriate for consideration of thermodynamical problems) and **(2)** the principle generally and roughly speaking expressing tendency of increasing the entropy in closed systems. All other its statements are deducible from these two principles by means of application of the differential and integration calculus. The second principle of the phenomenological thermodynamics shows that the processes studied in it are in principle irreversible while the processes described by means of the laws of classical mechanics are reversible. Besides, studying the processes in a given system the phenomenological thermodynamics is interested only in the initial and final conditions of the system treating it in the both conditions as being in equilibrium.

4.1.2.1.2.3) In a paper entitled “Illustrations of the dynamical theory of gases” presented 21.09.1859 and published during the next year James Clerk Maxwell (1832-1879) proposed a kinetic model of the gas and on its basis – a law of distribution of the velocities of gas-molecules. Proceeding with this direction of research mapped out by Maxwell, since 1866 with the article entitled “Über die mechanische Bedeutung des zweiten Hauptsatzes der mechanischen Wärmetheorie” (in English: On the mechanical meaning of the second principle of mechanical theory of heat”) Ludwig Boltzmann (1844-1906) began systematic elaboration of the kinetic theory of matter in which following the atomistic tradition and combining the laws of classical mechanics

with statements of probability theory he succeeded to obtain a noumenological (respectively noumenal, i.e. by means of revealing of the internal mechanism of the studied phenomena proposing in the same time an explanation of them) statistical interpretation of the concept “entropy” and in such a way also of the second principle of phenomenological thermodynamics. So, on the basis of ideas of kinetic theory of matter the thermodynamic was developed in a form of statistical thermodynamics (parallel to the phenomenological thermodynamics).

4.1.2.1.2.4) The formation of thermodynamics as a science in a historical process from Carnot to Boltzmann and Gibbs influenced the further development of scientific knowledge in various directions: **(1)** it was a starting point for further development of the thermodynamics during 20th century, **(2)** the elaboration of kinetic theory of matter and of statistical thermodynamics plaid the role of a starting point for subsequent laying down the foundations of statistical mechanics by Boltzmann and Gibbs, **(3)** it found essential applications in chemistry and in such a way it contributed to development of the last one, **(4)** it strengthened the theoretical (scientific) basis of engineering and especially of the heath engineering, **(5)** it reflected on some very important discussions in the epistemology and methodology of the scientific knowledge – the success of phenomenological thermodynamics demonstrated the possibilities of phenomenological approach to scientific problems, besides the success of kinetic theory of Boltzmann demonstrated in the same time success of noumenal approach to scientific problems as well as of the atomic conception in physics.

4.1.2.1.3) *Formation of electrodynamics*

4.1.2.1.3.1) After accumulation of a good deal of empirical information about electricity and magnetism during the 18th century and the first half of 19th century as well as after a considerable and much more prolonged since the time of Roger Bacon (1214?-1294) development of optic, Maxwell in the period of time 1855-1873 laid down the foundations of electrodynamics. Like Newton’s historical foundations of mechanics Maxwell’s historical foundations of electrodynamics **(1)** is constructed on the grounds of a few basic laws, namely of four ones, **(2)** has performed an integrative function, namely it has unified the problems concerning electricity, magnetism and light, **(3)** the concept of force plays a significant role, **(4)** the laws of physical phenomena are expressed by means differential equations which can be treated as demonstration of triumph of the idea of continuity in physics. In the same time, quite naturally, they differ from Newton’s foundations in many respects among which only two will be mentioned below: **(1*)** the type of forces treated in the electrodynamics is

different from the one in Newton's mechanic – they are connected not with the mass of a material body however with a specific substance – electric charge, besides parallel with the forces of attraction there are also analogical forces of repulsion, and (2*) despite of his declarations that the electrodynamics deals with new type of specific, very complicated physical processes, the phenomenological treatment of the topic of research realised by means of pure formal, mathematical derivation of the various physical propositions after all from the four basic laws mentioned above in the item (1) is combined in the scientific investigations of Maxwell with attempts to construct and use for explanation of the studied physical phenomena mechanical models, especially the one of ether as the material medium characterised by mechanical properties in which are realised the electromagnetic waves.

4.1.2.1.3.2) The electrodynamical theory of Maxwell seemed to the physicists at that time as very strange theoretical construction and its appearance provoked doubt of its correctness; so, it was generally accepted only after the death of its creator. After its acceptance however it became starting-point of various directions of further investigations among which only following three will be mentioned below: **(1)** some physicists attached great importance to Maxwell's ideas about the ether and made efforts to their improvement, **(2)** atomistic conception penetrated also in the theory of electricity: J.J. Thomson discovered the electron – the first elementary particles, **(3)** the Maxwell's electrodynamics became theoretical basis for important practical applications, on the first place the telegraph and radio were invented.

4.1.2.2) *Formation of chemistry as a natural science*

Note to item “4.1.2.2)” of the fact 14 in section 0.2.2.1.1. In the discourse below in the present item as well as in the following items concerning chemistry will be used especially [12].

4.1.2.2.1) Chemical knowledge on a very elementary level and in a very primitive form in immediate connection with requirements of the practical problems of life was developed in the course of practical action of the mankind since the hoary antiquity. It obtained systematisation to a certain extent in the framework of ancient Greek philosophy, especially in the philosophy of Aristotle, by means of the teaching about four basic elements composing all material things – earth, water, air, fire. During the Middle Ages at first in the Arabian world and later in the West Europe the chemical knowledge obtained self-dependent theoretical form as *the alchemy*, based on erroneous principles starting from just mentioned teaching about the four elements, being a mixture of truth and errors, so motivating misdirected aims of experimental

investigation, however nevertheless notably enriching it. These historical phases of development of chemical knowledge just mentioned constitute *the pre-scientific stage* of development of the chemistry during which the prerequisites for origination of chemistry as a science were created.

4.1.2.2.2) The very process of formation of chemistry as a science initiated during the Renaissance and later step by step reached completion during the second half of 19th century.

4.1.2.2.2.1) *The first step of process of formation of chemistry as a science.*

During 16th and 17th centuries the investigations of chemical phenomena were carried on to an increasing degree especially in connection with practical problems, mainly in the framework of medical and metallurgic research, often starting from the principles of Aristotle and of alchemy.

4.1.2.2.2.2) *The second step of process of formation of chemistry as a science* was carried out during second half of 17th century and first decades of 18th century. In connection with the requirements of the developed manufacture at that time of special interest become the problems concerning the phenomena of burning, or combustion, and in accordance with the prevailing general philosophical understandings of this time the theory of phlogiston was used as a way of their explanation. Nevertheless that later its groundlessness was proved, at that time it plaid a positive role for unification and development of chemical knowledge.

4.1.2.2.2.3) *The third step of process of formation of chemistry as a science.* During the middle and second half of 18th century the chemical experiments were considerably improved and the basic method of research in chemistry became the qualitative and quantitative chemical analysis. This results into refutation of the theory of phlogiston and discovery the oxygen by means of which the phenomena of burning, or combustion, obtained a satisfactory explanation from a scientific point of view and this event stimulated considerably the further development of chemistry.

4.1.2.2.2.4) *The fourth step of process of formation of chemistry as a science.* Nevertheless that during 18th century the atomistic natural-philosophic conception, which historical roots are in the ancient Greek philosophy, is very popular, the atomistic conception in chemistry was gradually formed in the course of scientific investigations of concrete chemical problems by Dalton during the period of time 1793-1808. The atomistic ideas of Dalton, which subsequently obtained further elaboration, opened a new very important direction of the historical development of chemistry.

4.1.2.2.2.5) *The fifth step of process of formation of chemistry as a science* consists in **(1)** a considerable enlargement of the scope of chemical research as a result of origination of the organic chemistry during the first quarter of 19th century and **(2)** subsequent synthesis of organic substances in the chemical laboratories (i.e. not in the living organisms) immediately demonstrating in such a way the unity of inorganic and organic substances.

4.1.2.2.2.6) *The sixed step of process of formation of chemistry as a science*, which focused various lines of the former progress of chemical knowledge and in which just mentioned process of formation of chemistry as a science reached its completion, consists in **(1)** the discovery by Mendeleev of periodic law of chemical elements founded on the atomic weight and **(2)** the composition on its basis of the periodic table of chemical elements, in such a way visually expressing it, in 1869. This periodic law embodied in the periodic table of chemical elements reveals *fundamental deterministic relations* between last ones, i.e. between the substantial elements of the chemical phenomena, allowing in such a way deduction in pure rational way and with some approximation the properties of chemical elements depending from their place in the periodic table. In consequence of this the periodic law embodied in the periodic table of chemical elements turned out to play in chemistry a foundational role, which is analogical to the foundational role of axiomatic systems in mathematical theories: it ensured a rational unity of the various elements of chemistry initially obtained by means of experience giving the chemical knowledge the form of scientific theoretical system and in such a way it manifested the completion of the process of formation of chemistry as a science. Besides this completion turned out to be not an end of historical development of chemical knowledge at all, but to be bases for a further improvement of the chemistry as a science.

Indeed on the basis of the periodical law Mendeleev made prediction of several unknown at that time chemical elements and of their properties which soon were fulfilled and namely this success of his theoretical constructions resulted into acceptance of his periodical law and table. It is worthy to be stressed that this fundamental contribution into chemistry was accomplished namely in result of effort of Professor Mendeleev during preparation of textbook entitled “Foundations of chemistry” (originally in Russian: „Основы химии”) to introduce a logical order in the exposition of properties of various chemical elements.

4.1.2.2.2.7) In perspective connection with the mentioned above in item 4.1.2.2.2.6 it is worthy to be mentioned here another line of development of chemistry during 19th century: the formation of physical chemistry as an

interdisciplinary field between physics and chemistry the subject of which is the interconnection between physical and chemical phenomena and which in some degree proposes physical theoretical foundations of the chemical phenomena.

4.1.2.3) *About formation of biology as a natural science*

4.1.2.3.1) (1) After accumulation **(1.1)** a good deal of various kinds of new information **(1.1.1)** about **(1.1.1.1)** plants and **(1.1.1.2)** animals, including about homo sapience **(1.1.2)** based on **(1.1.2.1)** systematic observations (also using dissections of bodies of various species including the homo sapience) and **(1.1.2.2)** experiments – **(1.1.2.2.1)** artificial ones, as well as **(1.1.2.2.2)** ones carried out under natural conditions of the practical activities in the fields of **(1.1.2.2.2.1)** the agriculture and **(1.1.2.2.2.2)** the medicine **(1.2)** during 16th -18th centuries as well as at the beginning of 19th century **(2)** two general fundamental conceptions about the study of life were elaborated during 19th century, **(2.1)** exposed in the following two treatises - **(2.1.1)** “Philosophie zoologique” (1809, “Philosophy of zoology”) by Jean Baptiste Lamarck (1744-1829), and **(2.1.2)** “The origin of species” (1859) by Charles Darwin (1809-1882), **(2.2)** completing the process of formation of the biology as a science of life (initiated by Aristotle).

4.1.2.3.2) *Lamarck’s treatise “Philosophie zoologique”*

4.1.2.3.2.1) The treatise “Philosophie zoologique” combines fundamental philosophical and zoological (with general biological significance) ideas and approaches in such a way that it turns out to be a considerable event in the both historical fields – in the history of philosophy as well as in the history of biology. At the very outset of the treatise Lamarck pointed out that the philosophy of zoology is a collection of principles and based on the last ones rules concerning the study of animals, besides they are applicable also to other branches of the natural science.

4.1.2.3.2.2) Starting from very general philosophical statements and approaches expressing a tendency **(1)** towards concord and unity in the theoretical scientific reasoning between **(1.1)** relying on thorough empirical grounds, which is connected with application of induction, and **(1.2)** formulating of and starting from general principles, which is connected with application of deduction, as well as **(2)** towards application in the treatment of problems of special sciences, at the given case of biology, of the philosophical principles **(2.1)** of material unity of the nature, **(2.2)** of the determinism, and **(2.3)** of combination of **(2.3.1)** exact phenomenological description of the studied phenomena, and **(2.3.2)** construction of

noumenological hypotheses for explanation of the studied phenomena by means of revealing their internal (beyond the empirical data) mechanism and causes, **(1→2)** step by step he passes (especially in accordance with the mentioned in the subitem (1) above) from them to relatively more concrete biological problems and statements which are possessed of general methodological significance for the biology as a whole.

4.1.2.3.2.2.3) The “very general philosophical statements and approaches” mentioned right from the start of item “4.1.2.3.2.2)” as well as the principles of zoology, formulated or merely expressed in the text of “Philosophie zoologique”, are elaborated in the course of scientific researches by Lamarck during his lifetime also in accordance with that which was mentioned above in item “4.1.2.3.2.2)” (1), however in the opposite order in respect to the exposition in the treatise: they were formed in the course of long, thorough and fruitful investigations in the special biological fields of botany and later of zoology, in particular on the problem of classification in the both fields, as well as in the fields outside biology, namely in the field of geology and in the interdisciplinary field between geology and biology - the field of palaeontology.

4.1.2.3.2.4) In the three parts of “Philosophie zoologique” Lamarck has elaborated and expounded correspondingly three general fundamental biological conceptions of principal, methodological character for biology as a science, intended to unify its various detailed parts:

4.1.2.3.2.4.1) *a theory of biological evolution*, i.e. of a natural process step by step to develop higher animals from lower ones **(1)** elaborated in close connection with the treatment of very actual at that time problems of classification of animals, **(2)** essentially based on the acceptance of the thesis about the principal role of using or not using of an organ of the given animal in order to meet its new needs imposed by the new circumstances in the environment for its improvements or, on the opposite, for its decay and even for its full vanishing, respectively, and **(3)** logically concluded with short formulated conception about the origin of homo sapiens;

4.1.2.3.2.4.2) *a conception about the physical determination of the processes of life* in the organism which is explained by means of mechanism based on the concept of fluids penetrating into the organism from the surroundings; and

4.1.2.3.2.4.3) *a psycho-somatic theory* **(1)** in which an attempt has been made to reveal the physical causes and mechanism of the psychic processes, mainly of the cognitive ones, **(2)** starting from a general philosophic position

that the physical and the psychic possess a general reason and (3) again using the concept of fluid, besides introducing its specification “nerve-fluid”.

4.1.2.3.2.5) (1) In accordance with his philosophic-methodological view expressed at the outset of his treatise and anticipated the later formatting positivism that in the knowledge only the well established facts are reliable besides the theoretical constructions require critical and even sceptical attitude **(2)** as well as obviously taking into account that his attempt to reveal *the causes and the internal mechanism* of the processes of life (which is moreover pioneering one) is connected with the risk to turn out to be mistaken, **(3)** at the end of the treatise Lamarck has stressed his hope that nevertheless some errors might be found in his fundamental work the last one should be useful for further progress of the scientific knowledge.

4.1.2.3.2.6) A factor for such a large-scale and simultaneously very successful research of Lamarck on relatively more concrete, special scientific problems before composition of “Philosophie zoologique” mentioned in the item “4.1.2.3.2.2.3)”, which has played an important preparatory role for formation of the principles and very general and fundamental statements in the field of philosophy and zoology formulated or merely expressed in the text of just mentioned treatise, together with the personal intellectual abilities of its author was also that his work proceeded in the presence of a very good intellectual environment stimulating his work - three famous naturalists are worthy to be mentioned in this connection: **(1)** young Lamarck was friends with one of the most popular biologists at that time Georges Buffon (1707-1788), **(2)** the way of his research was interlaced with the way of research of one of greatest naturalists at that time George Cuvier (1769- 1832), **(3)** he was in contact and even has undertaken joint actions with the noted zoologist Geoffroy Saint-Hilaire (1772-1844). However Lamarck’s treatise was often underrated and ultimately it obtained inadequate interpretation: usually its very rich of problems and ideas content was reduced only to the theory of evolution, besides his explanation of its mechanism (cf. item (1.2) above) obtained well-grounded rejection - it was overcome in an categorical way by the Darwin’s theory of evolution exposed in his treatise “The origin of species”.

4.1.2.3.3) Darwin’s treatise “The origin of species”

4.1.2.3.3.1) The treatise “The origin of species” by Darwin was published a half of century after Lamarck’s treatise “Philosophie zoologique” and during this interval of time a considerable amount of empirical information relevant to the problems of biological evolution was accumulated. In such an actual situation and moreover after elaboration of the philosophic-methodological

principles of biological conception of evolution in “Philosophie zoologique”, which Darwin has actually accepted and applied in his proper research, in distinction from Lamarck’s treatise, “The origin of species” was entirely devoted to the topic of biological evolution as a subject of special biological investigation – it was composed and published as work in a special, particular field of science, namely as a biological work, not as a philosophical one. Besides, while in “Philosophie zoologique” three fundamental topics of biology are treated (cf. item “4.1.2.3.2.4”), the treatment in “The origin of species” is essentially restricted only with one of these topics – the topic of biological evolution.

Note to item “4.1.2.3.3.1)” of the fact 13 of section 0.2.2.1.1. This relationship between Lamarck’s “Philosophie zoologique” (1809) and Darwin’s “The origin of species” (1859) during 19th century in many respects resembles the relationship between Descartes’ “Principia philosophiae” (1644; authorized translation into French: “Les principes de la philosophie”, 1647) and Newton’s “Philosophiae naturalis principia mathematica” (1687) during 17th century.

4.1.2.3.3.2) (1) Nevertheless of the fact mentioned above in item “4.1.2.3.3.1)”, that the text of “The origin of species” is strongly restricted to treatment of biological problems concerning evolution of species, and so there is no place in it for explicit considerations of philosophical problems, **(2)** owing to the circumstance that the “narrow, special biological topic concerning evolution of species” is **(2.1)** of very complicated character, as well as **(2.2)** of fundamental, principle importance for investigations **(2.2.1)** of various problems in the field of biology, as well as **(2.2.2)** of various scientific problems outside of biology (as for example in the field of geology and palaeontology), **(3)** the way of treatment of this topic in the book has manifested a definite approach to scientific investigations, and so it has expressed implicitly, however very strikingly, philosophic-methodological principles underlying the given research and in such a way is possessed of universal significance for the scientific knowledge and investigations at all. One can show in particular following philosophic-methodological principles of scientific research realised in the way of elaboration of the theory of evolution exposed in Darwin’s “The origin of species”: **(1)** unity of various forms of scientific research - unity **(1.1)** of empirical and rational investigations, **(1.2)** of induction and deduction, **(1.3)** of phenomenological description of the studied phenomena and there noumenological explication (namely which in the given case attaches cogency to the proposed theory of evolution), **(2)** aspiration **(2.1)** for revealing internal connections between various parts and aspects of nature and further for comprehending their (as well as: its) unity **(2.2)** by means of treatment of natural entities not as absolutely steady and as being at a standstill however as

being in a process of change and development **(3)** resulting into a tendency of revealing internal connections between various natural sciences and further of comprehending their unity, respectively of comprehending the unity of the natural science.

Moreover, owing to the character of the topic of Darwin's research his theory of evolution has consequences for the world outlook, especially by means of treatment of the last chain of the evolution in "The Descent of Man" (1871), hence it is relevant to the philosophy of life.

4.1.2.3.3.3) On the basis of a great amount of empirical facts in "The origin of species" Darwin has developed systematically and in details a theoretical construction of the process of origin of species in which one may reveal following eight basic factors for the realisation of biological evolution:

4.1.2.3.3.3.1) variability of the organisms,

4.1.2.3.3.3.2) inheritance at least of some variations,

4.1.2.3.3.3.3) conditions of life of the environment of given organism,
which are also variable in the course of time,

4.1.2.3.3.3.4) great quickness of the reproduction of organisms,

4.1.2.3.3.3.5) struggle for existence (as a consequence of the factor
mentioned in previous item "4.1.2.3.3.3.4)),

4.1.2.3.3.3.6) survival of the fittest beside the unfitted organisms perish,

4.1.2.3.3.3.7) natural selection which is a result of the joint action of the
previous six factors,

4.1.2.3.3.3.8) long-term process of the global life in the course of time
realised by means of life of many successive generations,
otherwise speaking, long-term joint action of the factors
mentioned in items "4.1.2.3.3.3.1)"- "4.1.2.3.3.3.5)".

Otherwise speaking, the evolution of the life on earth is result of long-term action of the mechanism of natural selection affecting great amount of successive generations and consisting in varying the characteristic features of organisms, besides at least some of the variations are hereditary, and in the struggle for existence under given variable conditions of life of the environment, resulting from great quickness of the reproduction of organisms, the fittest survive and unfitted organisms perish. The long-term process of accumulation step by step of those variations which correspond to the conditions of the changeable environment results into considerable modifications of the organisms and ultimately into origin of new species,

besides the intermediate forms between various species have perished as unfitted. On the bases of this theoretical treatment of origin of the species various phenomena which we know by experience obtain their convincing explanation which serve as its additional corroboration.

4.1.2.3.3.4) The time of composition of “The origin of species” (1859) was very auspiciously for elaboration and proposition of a convincing, empirically well grounded evolutionary theory in biology: **(1)** it appears a half century after Lamarck made the first attempt for its detailed theoretical development in which he **(1.1)** proclaimed substantial philosophic-methodological principles for its elaboration as well as **(1.2)** outlined some right directions of the thought for research on the given topic, and **(2)** the achievements of geology, as well as of palaeontology on the boundary between geology and biology, proposed essential indirect arguments in favour of evolutionary conception in biology. “The origin of species” were created under these favourable conditions as a result of many years investigations by Darwin, proceeded in well ordered successive stages **(1*)** from five years long empirical study - observations of various forms of life as well as geological observations – during circumnavigation on the surveying ship “Beagle” and successive experiments with pigeons, **(2*)** through thorough analysis of the accumulated empirical stuff, to **(3*)** systematic exposition of the theory of biological evolution. His scientific work was supported by many colleagues among which, Charles Lyell (1797-1875) - the author of famous “The Principles of Geology” (1833), exerted essential influence on development of the evolutionary conception in biology.

4.1.2.3.3.5) The appearance of “The origin of species” at first provoked some negative reactions mainly in consequence of discrepancy between exposed in it evolutionary theory and widely spread world outlook, however after much debate Darwin’s theory of evolution was accepted in biology and additionally substantiated. Moreover a lot of biological problems obtained their decision on the basis of evolutionary theory as well as a lot of biological statements became clearer and better argued when they were elucidated from its point of view. So, Darwin’s theory of evolution turned out to be a factor **(1)** unifying various biological facts into a unified theoretical system of biology and in such a way the historical process of formation of biology as a science, initiated at first by Aristotle, was completed. From a theoretic-constructive point of view this means that Darwin’s theory of evolution played in the history of biology a role similar to the one of Euclid’s axiomatic system in the history of mathematics: indeed one can point out in this respect a significant analogy between relations “the species **B** is descended from species **A**” in biology, on the one hand, and “the proposition **B** is derivable from proposition **A**” in logic, on the other hand.

4.2) *The process of development of the classical mathematics*

4.2.1) Almost during the same time of decay of the Greek mathematics after Diophantus (cf. fact 12, item “3”) a rise of Arabian mathematics initiated. The last one was developed by means of adopting the mathematical achievements from two sources: Greek and Hindu ones. During 12th and 13th centuries Europe was in contact with Arabs and adopted their knowledge in mathematics. In such a way through the mediation of Arabs Europe take acquaintance with the ancient Greek mathematics as well as with the Hindu mathematics which was quite different from the Greek one: while in the Greek mathematics dominated geometry, so that the arithmetic and algebraic problems were treated using geometric language, in the Hindu mathematics on the contrary dominated arithmetic, i.e. the concept and calculation of numbers. So, **(1)** such a combination of influences of both kinds of mathematics, Greek and Hindu, respectively of both traditions in it - geometric and arithmetic, besides realised under **(2)** the new conditions of formation and development of the classical natural sciences (cf. this factor 14, item “4.1”), which deal mainly with processes and accordingly search for laws of the studied phenomena by means of quantitative, numerical analysis of them **(1→2)** turned out to be very auspicious circumstance for a new up-lift of the European mathematics.

4.2.2) In the course of very intensive development of mathematics during the period of time from the middle of 15th c. till the boundary between 19th and 20th centuries, more precisely speaking till 70-thies years of the 19th century (i.e. till beginning of elaboration of the Cantor’s set theory and publication of Frege’s “Begriffsschrift”, 1879), mathematics was not only enriched with a new content, i.e. with an enormous amount of new theorems, however also its shape, i.e. its form as a science underwent modifications in the framework of the implicitly expressed in Euclid’s “Elements” conception of mathematics as a science about geometric figures and numbers, the propositions of which about last ones are either preliminary accepted as definitions, postulates, common notions, or are logically proved ultimately on the bases of the first ones. Below following three kinds of modifications of the form of mathematics as a science will be mentioned as very significant:

4.2.2.1) Modification of treatment of the subject-matter of mathematics

This type of modification can be subdivided into following two kinds:

4.2.2.1.1) *Modification of relation between both components of the treated in such a way (as expressed in Euclid’s “Elements”) subject-matter of mathematics: figures and numbers*

The understanding of subject-matter of the mathematics as consisting of numbers and figures, otherwise speaking the treatment of mathematics as a science of quantitative and spatial relations (or: of quantitative relations and spatial forms), implicitly was held during this period of time, besides during 19th century it was even expressed explicitly in various variants by different authors. In the same time however a profound alteration of the treatment of relation between both components of the subject-matter of mathematics (i.e. of the treatment of relation between numbers and figures, between quantitative and spatial relations) during this period of time took place. Following five steps of the process of change of this relation will be mentioned below.

4.2.2.1.1.1) First step. As early as during the first phase of the development of mathematics of the period of time which is under consideration here, embracing the second half of 15th c. and the 16th c., quite likely under the influence of Hindu mathematics, algebraic problems, mainly concerning solution of algebraic equations in a proper (i.e. in a non-geometric) form, were subjected to very intensive and enthusiastic investigations – let to be mentioned the work of Lucas Pacioli (somewhere in the middle of 15th c. – 1510), Nicholas Tartaglia (1500-1557), Girolamo Cardan (1501-1576).

4.2.2.1.1.2) Second step. A process of including irrational numbers in mathematics initiated during the first phase mentioned above by means of innovation of the term denoting them – “irrational numbers” – by Michael Stifel (1486-1567) in 1544. Subsequently the irrational character of some very important constants was proved and step by step the theory of irrational numbers was developed. As a result of these achievements, in opposite of the ancient Greek mathematics, in particular in opposition of Euclid’s “Elements”, already there is no reason to present the irrational magnitudes in a geometrical way, namely by means of lines. Moreover, other kinds of numbers were introduced in mathematics as negative, complex, hyper-complex ones and this circumstance resulted into *reinforcement of the role of calculations in mathematics*.

4.2.2.1.1.3) Third step. During the same first phase mentioned above the application of symbols to presentation of arithmetic and algebraic expressions increased more and more and ultimately obtained a systematic form in the François Viète’s (1540-1603) work “In Artem Analyticam Isagoge” (“Introduction to the Art of Analysis”, 1591). This work was historically the first step towards transformation of the wordy (or: verbal) algebraic calculus, allowing only more or less however in any case only modest using of symbols, into formal algebraic calculation and in such a way it was historically the first step towards transformation of the wordy (or: verbal) algebra into symbolic algebra (as well as of the wordy arithmetic into symbolic arithmetic).

Subsequently step by step the symbolic language of algebra, i.e. the language of algebraic equations, was enriched and improved, and it became a powerful tool for further development of arithmetic and algebra, as well as of their applications. Moreover, its general idea exceeds the limits of arithmetic and algebra and was developed also as artificial language of various new forms of calculi - of differential and integral calculus, calculus of probabilities, logical calculus of propositions.

4.2.2.1.1.4) Fourth step. During the second phase of the stage of development of classical science, embracing the period of time since beginning of the 17th c. till the beginning of mathematical research of Newton and Leibniz, i.e. till about 1665, an important process of formation and development of the analytical geometry initiated by means of publication of Descartes's "La Géométrie" (1637). By means of invention of the analytical geometry the way of treatment of mathematical problems underwent radical change: while in the ancient Greek mathematics dominated the geometrical way of treatment of problems about numbers, i.e. chiefly of arithmetical and algebric problems, now, after introducing in mathematics irrational numbers and formal symbolic language (cf. items "4.2.2.1.1.2)" and "4.2.2.1.1.3)"), in the analytical geometry on the contrary the geometrical situations and problems obtain formal symbolic expression by means of algebraic equations (or algebraic inequality) and so the process of solution of geometrical problems was reduced in a considerable degree to algebraic calculus. This method of treatment of geometrical problems together with using the concept of coordinate system (which is basic one in its framework) turned out to be very fruitful for the development of geometry itself, besides, however, the meaning and impotence of analytical geometry exceeded the limits of geometry: providing the formal symbolic expressions of the algebra with visual geometrical models it turned out to be very fruitful also for development of the algebra, in particular for elaboration of the vector algebra later, as well as it turned out to be a fundamental preliminary condition for the invention of the differential and integral calculus.

4.2.2.1.1.5) Fifth step. The main direction of the development of mathematics during this stage of its history – during the period of time from the middle of 15th c. till the boundary between 19th and 20th centuries, more precisely speaking till 70-thies years of the 19th century (i.e. till beginning of elaboration of the Cantor's set theory and publication of Frege's "Begriffsschrift", 1879) – consisted in the invention of the differential and

integral calculus by Newton (1642-1727) and Leibniz (1646-1716) during the second half of 17th century and afterwards in the many-sided development of the mathematical analysis by their followers. The invention and further

development of mathematical analysis was motivated mainly by the requirements of the development of geometry of the curves and of surfaces containing curves, on the one hand, and the requirements of the development of mechanics as a science of the various kinds of movements of material bodies in the space and time, on the other hand; in the same time it was prepared by the all four former steps mentioned in the items “4.2.2.1.1)” - “4.2.2.1.1.4)”, respectively. Simultaneously, namely in the course of successful applications of mathematical analysis in the field of geometry, mechanics, as well as in other branches of physics, it obtained additional stimulus and direction for further development.

4.2.2.1.2) *Going out of the widely accepted treatment of subject of mathematics in the course of mathematical research*

A discrepancy between the usually accepted (at least implicitly) understanding of subject-matter of the mathematics as consisting of numbers and figures, otherwise speaking the treatment of mathematics as a science of quantitative and spatial relations, on the one hand, and the real choice of the concrete subjects of mathematical investigations, on the other hand, sometimes took place during this period of the history of mathematics mentioned above – in some cases the subjects of mathematical investigations turned out to be neither figures, i.e. spatial relations, nor numbers, i.e. quantitative relations, in a pure form. As examples let be mentioned: **(1)** theory of group, **(2)** Boolean algebra, **(3)** algebra of logic, **(4)** probability theory (indeed in the last two cases numbers are used, but not in pure form however as measures of qualitatively defined entities).

4.2.2.2) *Modifications of the logical form of mathematical theories*

4.2.2.2.1) The propositions forming the content of a given mathematical theory were usually descriptive, not prescriptive, as it was in many cases in the ancient Greek mathematics as it is presented in Euclid’ “Elements” (cf. this section, item “3.1)”).

4.2.2.2.2) The distinction between two kinds of postulated propositions in the axiomatic system of mathematics exposed in Euclid’s “Elements” – **(1)** “postulates” and **(2)** “common notions” (cf. “3.2)”), is abolished, so that the axiomatic systems in mathematical theories during this stage of the history of mathematics consists only of one type of postulated propositions – just of axioms.

4.2.2.3) *Initial steps towards a deep modification in the nature of mathematics as a science leading up to origination of foundations of mathematics*

4.2.2.3.1) Along with rapid growth of new results in mathematics during 17th, 18th and first three quarters of 19th century also seeds of new points of view on the mathematics as a science appeared which during the next stage of its history blossomed out into deep change in its nature, resulting in particular into formation of a new, very peculiar field of mathematical knowledge – foundations of mathematics.

4.2.2.3.1) Descartes during the first half of 17th century and Leibniz during the second half of 17th century and at the beginning of 18th century, being simultaneously mathematicians and philosophers, had proposed a philosophical look at mathematics from which subsequently one direction of the philosophy of mathematics arose. Leibniz suggested ideas about connection between mathematics and logic which at the end of 19th century and during the first half of 20th century were developed into the conception of logicism; Kant in opposition to Leibniz developed a tendency of treatment of mathematics coming from Descartes which were realised afterwards during the 20th century in the intuitionism. So, the debate about the problem concerning relationship between mathematics and logic (including as an essential element of it the problem concerning relationship between formal logical procedures, on the one hand, and the intuition, respectively the contentful (i.e. with regard to the contents) aspect of the process of thinking, on the other hand, turned out to be of fundamental importance for treatment of the nature of both mathematics and logic.

4.2.2.3.2) In one of the fundamental part of mathematics at that time – geometry, deep structural change took a place: together with the Euclidean geometry different other geometrical systems arose during 17th, 18th and about first half of 19th century as projective geometry - Desargues (1593-1662), Pascal (1623-1662), Poncelet (1788-1867), affine geometry – Möbius (1790-1868), non-Euclidean geometries – hyperbolic and elliptic – Saccheri (1667-1733), Gauss (1777-1855), Lobatshevsky (1793-1856), Bolyai (1802-1860), Riemann (1826-1866), and so on. In view of this situation a task to put in order these different geometrical systems (respectively, these different geometries) and in such a way to unify them in a very complicated system in the framework of the unified geometry was raised. This task was distinctively manifested in 1772 by the successful attempt of Felix Klein (1849-1925) in his famous Erlangen programme of proposing a solution of it by means of application of the young very abstract branch of algebra, namely of the theory of group, to the geometrical problems.

4.2.2.3.3) From the Renaissance till the 19th century mathematics was in a process of rapid development – new mathematical problems, new approaches

to their treatment, new mathematical theories appeared one after another besides the attention of mathematicians was directed chiefly to obtaining new results, in particular of such a kind which are satisfying the requirements of the physical sciences and technology, and usually too little to satisfying the requirements of logical exactness of the formulations of statements and rules and of rigorous logical ways of the proofs of theorems. In such a case in some parts of mathematics the intuition and experience turned out to be enough to avoid making mistakes. However in the field of mathematical analyses, i.e. in the field of calculation with the infinity, where the potentiality of the intuition is considerably limited, confusions and contradictions appeared and in such a way namely in the leading branch of mathematics at that time besides using most powerful and effective influence on the development of natural sciences and technology an intolerable situation took place. In response to the last one Augustin Cauchy (1789-1857), Karl Weierstrass (1815-1897) and others outstanding mathematicians made efforts to construct the building of mathematical analyses logically in an irreproachable way. This work historically initiated the process of foundational studies (in German: Grundlagenforschung) in modern mathematics which was completely realised during the next stages of the history of mathematics. Indeed, later, David Hilbert at the very outset of an address-article, outlining his programme of foundations of mathematics has declared: “Weierstrass ...created a firm foundation for mathematical analysis” ([15], p. 369), and later he almost repeated ”Weierstrass provided a foundation for the infinitesimal calculus” [Ibid., p. 369], “Weierstrass ... is establishing the foundations of analysis” [Ibid., p. 370].

The implementation of this portion of foundational studies concerning mathematical analysis has revealed however that the logically irreproachable construction of its building requires as a necessary condition this construction to be realised on the logically irreproachable foundations of the theory of real numbers. So, the problem of substantiation of the mathematical analysis was reduced to the problem of substantiation of the theory of real numbers, which becomes an actual problem for thorough investigation.

On the other hand the construction of non-Euclidean geometries, which in distinction from the Euclidean geometry did not extract any of their statements from the experience, raised the question for their consistency too.

4.3) *Concerning the process of formation of the philosophy of science*

4.3.1) The process of formation of classical science, or speaking otherwise, of various classical branches of science, as classical mathematics, classical physics, classical biology, was not merely a process of *quantitative increase* of knowledge, not merely accumulation of new knowledge to the old one, but on the first place it was a process of *qualitative* reconstruction of its system involving search of appropriate new ways and tools of scientific investigations, mainly of appropriate new (1) concepts, (2) principles and (3) methods to posing and solution of scientific problems. The treatment of these methodological problems, nevertheless in explicit or implicit form, brings up various epistemological and ontological questions for substantiation one or other methodological choice of the approach to investigation of the scientific problems. So, in such a way the elaborated ideas concerning the ways of knowledge in the framework of philosophy reflected on the form of scientific research and by means of the success or failure of the last one they obtained their corroboration or refutation, resulting into their flowering or falling into oblivion, respectively.

4.3.2) Parallel to the process of development of science and arts a process of decisive delimitation between two classes of problems, which in former times were interlaced in the philosophical thought – (1) problems connected mainly with scientific knowledge (even in the periods of time when the scientific problems and ideas were in rudimentary form), and (2) problems connected mainly with the problems of the way of personal life – has taken place. During 19th century this process has grown into initiation of a process of delimitation between two different types of philosophy, or, speaking otherwise into initiation of a process of split of the unique philosophy of a traditional sense of the word into two philosophies in different sense of the word: (1*) *philosophy of science*, which with its (1*.1) content of concepts, problems and propositions, and (1*2) analytical form of their treatment, (1*.1→1*2) responds in one or other degree to the specific methodological requirements of the scientific research, and (2*) *philosophy of life*, (2*.1) directed to discussion of vital questions of the man and societies (2*2) in a form which in one or other degree brings it near to the art.

4.3.3) The initial steps of the process of formation of philosophy of science, which is of special interest here, proceeds under following two types of conditions:

4.3.3.1) under conditions of competition and discussions between various philosophical tendencies, directions, schools – (1) empiricism and rationalism, (2) materialism (or in broadened sense: scientific realism) and idealism, (3) analytical versus synthetic approach, and so on;

4.3.3.2) under conditions of essential interconnections and even of interlacement between **(1)** researches in the field of philosophy and **(2)** researches in the field of various special sciences. In this connection it is worthy to be mentioned that **(1*)** during 17th c. and the beginning of 18th c. great philosophers as for example Descartes and Leibniz are also great names in the history of mathematics and of natural sciences; **(2*)** during the first half and about the middle of 19th c. – when the differentiation of sciences was going too far – a number of distinguished representatives of various special sciences published works demonstrating even by means of their title that they are devoted to consideration of philosophical problems of their special branch of science or of the science in general, treating actually some principle questions of the science, special and general, as for example J. Dalton’s “A new system of chemical philosophy” (1808), Lamarck’s “Philosophie zoologique” 1809, cf. item 4.1.2.2.2), André-Marie Ampère’s “Essai sur la philosophie des sciences, ou, Exposition analytique d’une classification naturelle de toutes les connessances humaines” (1834), Stanislao’s Cannizzaro’s “Sunto di un corso di filosofia chimica” (“Synopsis of a course on philosophy of chemistry”, 1858).

Fact 15.

At the boundary between 19th and 20th centuries (roughly the time-interval involving the last 20-25 years of 19th century and about the first 15 years of 20th century) deep changes in the science took place. In particular it appeared **(1)** discrepancy between the content of scientific theories and the results of experiment - in the natural sciences, **(2)** some kinds of logical contradictions, respectively paradoxes - in mathematics (together with mathematical logic), and **(3)** in the both fields of science mathematic and natural sciences – formation of essentially new ideas and new methods of research. So, the overcoming these difficulties and controversies as well as the further development of innovations just mentioned above turned out to be connected with the circumstance that an initiatory phase of cardinal reconstruction of the foundations of respective branches of science took place, resulted into acceptance of new languages (concepts), principles and ways of their systemic construction and in such a way accomplished a transition generally speaking from the classical science to non-classical science. The last ones are characterized in particular by means of the following two features: **(1)** the first steps of formation of new foundations of mathematics were realized, and **(2)** decisive steps towards preparation of formation of philosophy of science took place.

1) The process of making first steps to formation of the new foundations of mathematics was realized in following four lines:

1.1) The first line of the process of making first steps to formation of the new foundations of mathematics is connected with the work of Dedekind and Peano as direct continuation of the work of Weierstrass on foundations of mathematical analysis which had raised as a pressing problem the one concerning establishment of firm foundations of the theory of real numbers treated as a basis for construction of the imposing building of mathematical analysis (cf. item “4.2.2.3.3”).

1.1.1) Responding to the requirement of the day in connection with Weierstrass’s investigations on foundations of mathematical analysis mentioned above Richard Dedekind (1831-1916) elaborated a satisfactory theory of real numbers starting from the arithmetic of natural numbers exposed in a monograph significantly entitled “Was sind und was sollen die Zahlen” (“What are and what should to be the numbers”) (1888). So, the problem of foundations of mathematical analysis was ultimately reduced to the problem of foundations of the arithmetic of natural numbers.

1.1.2) During the next year after publication of Dedekind’s work mentioned above, namely in 1889, Giuseppe Peano (1858-1932) published a very short booklet entitled “Arithmetices principia, novo methoda exposita” (in English: “The principles of arithmetic, presented by a new method”) in which he actually continued the work of Dedekind on foundations of mathematical structures which play the role of presuppositions of mathematical analysis proposing a successful axiomatic system of natural numbers. In the same time in this booklet Peano has contributed considerably to the development of foundational studies (in Germany: Grundlagenforschung) proposing a new method in this respect which can be regarded as further improvement of the axiomatic method in direction of its formalization on the basis of the symbolic language of logic.

Note to item “1.1.2)” of the fact 15 in section 0.2.1. After introduction of logical notations as well as of notations about classes and functions in their general form and using the achievements of Boole and Schröder in the field of logic, Peano has introduced especially for the axiomatic system of natural numbers four signs – for “number (positive integer)”, “unity”, “successor of a ”, and “is equal to”, and further he formulated in symbolic form, using logical and set-theoretical relations nine axioms which can be treated in their totality as axiomatic definition of the concept “natural number” (respectively, “positive integer”). The last one (i.e. the treatment of the axiomatic system just mentioned as definition of the concept “natural number”) means that we treat the concept of natural number in pure formal way, namely as such an entity which satisfy the formulated axioms. Adding a scheme of definitions of the concrete natural numbers “2”, “3”, “4”, and so on, Peano has proposed formulation and offered proofs of a few theorems about natural numbers in symbolic logical form, without using any words of the natural language (however one can mention as defect

from contemporary point of view that these proofs are carried out without using in an explicit way any rules of inference).

In the contemporary mathematical literature Peano's axiomatic system of natural numbers is accepted in a simplified form: **(1)** four axioms are canceled from the list of original nine axioms (by the way for three of them this elimination is justifiably because they are of more general character – they define in their totality the concept “equality”, which applicability is not specific only for the natural numbers) **(2)** the rest five axioms often are formulated in the framework of stylized natural language, and as a consequence **(3)** the proofs of the various theorems on the basis if the axiomatic system is carried out without using the language and tools of proof of the mathematical logic. From the point of view of the author of present text this simplification of the original systems of axioms formulated by Peano is a step backward from his great achievements in direction to elaboration and application of strict methods for foundational studies in the field of concrete mathematical theory.

1.2) During a few last decades of 19th century some mathematicians as Kronecker and Poincaré were inclined to treat the system of natural numbers as the last basis of the hierarchic multiformity of various mathematical structures and correspondingly to see in the arithmetic of natural numbers – in its content and methods of research, the last foundation of mathematics. The second line of the first steps in the process of formation of the new foundations of mathematics is connected with making break in this view by means of invention of deeper structures on the basis of which a construction of the arithmetic of natural numbers was proposed. At the last two-three decades of 19th century and the very outset of 20th century two new mathematical theories were developed as new foundations of the arithmetic of natural numbers: **(1)** Cantor's set theory, setting new, more general and in the same time more abstract subject of mathematics than the numbers and figures (respectively, than quantitative and spatial relations) – namely arbitrary, abstract elements and their sets, besides using very powerful abstractions and methods of thinking in accordance with the epistemology of the realism, and **(2)** Frege's formalized in essence classical propositional and predicate logic, distinguished itself by essentially strengthening of exactness of logical inference, on which basis the arithmetic of natural numbers was constructed in order to give in such a way cogent argument in favour of Leibniz's philosophy of mathematics against the one of Kant (cf. fact 14, “4.2.2.3.1”). These both new directions of the mathematical thought obtained afterwards very dramatic way of further development - **(1*)** on the one hand in the both fields some kind of contradictions were discovered, so an intolerable situation appeared in them, and **(2*)** on the other hand for the both fields it turned out that they can serve as foundations not only of the arithmetic of natural numbers, however of the all mathematics (the corresponding directions in foundations of mathematics are called in the literature cantorism

and logicism, respectively). The simultaneous validity of both situations (1) and (2) mentioned above for the both set theory and logic raised the problem concerning their foundations, in particular to be ensured their consistency (i.e. that they are free from contradictions). It is worthy of mention in this connection many-sided, profound research on set theory and classical logic of predicates by Bertran Russell (1872-1969), especially in the fundamental three-volumes treatise written by him jointly with his teacher Alfred North Whitehead (1861-1947) “*Principia Mathematica*”, 1910-1913 (cf. [13]), devoted to systemic exposition of the set theory, logic and construction of fundamental mathematical structures on a logical basis.

1.3) Responding to the actual problem in the geometry concerning revealing the logical relations between different systems of geometry, i.e. between different geometries, and establishment in such a way of the unity of geometry (cf. fact 14, item “4.2.2.3.2”) by means of very skilful application of the axiomatic method in an essentially developed form in direction of reinforcement of abstract character of the axioms David Hilbert (1862-1943) in his monograph entitled “*Grundlagen der Geometrie*” (in English: “*Foundations of geometry*”) (1899) proposed a new axiomatic system of geometry which is more perfect in comparison with the one proposed in the ancient time by Euclid – it is essentially more abstract and structuralized, allowing revealing the logical relations between different geometries and in such a way to achieve their unification. A little later Hilbert applied the so improved axiomatic method to foundations of arithmetic and logic too (cf. [16] and [17]). The approach to foundational problems of geometry, arithmetic and logic in these three works mentioned above (1899, 1900 and 1904) represents the initial form of Hilbert’s direction in the foundations of mathematics called formalism and connected chiefly with requirements of systemic application of axiomatic method in a very abstract form, so that the words used in the formulation of axioms and theorems are devoid of proper conceptual sense.

1.4) Obviously in order to respond to the paradoxes discovered in the set theory, besides connected with the way of using of its basic concepts _ “set”, during 1908 Ernst Zermelo (1871-1953) has published axiomatic foundations of set theory in spirit of Hilbert’s early formalism in an article entitled “*Untersuchungen über die Grundlagen der Mengenlehre I*” (in English: “*Investigations on the foundations of set theory I*”). In such a way Zermelo has initiated the historical process of research on the axiomatic foundations of set theory which during the next stage of the history of mathematics enriched this field of mathematical investigations with other approaches to construction of axiomatic foundations of set theory and in the same time with other concrete (in

some sense) axiomatic systems of set theory resulting into formation of separate branches of the set theory – “abstract set theory”, “axiomatic set theory”, “foundations of set theory” (cf. [18] - [21]).

1.5) Since 1907 Brouwer (1881-1966) began elaborating a new direction in the field of foundations of mathematics which in contrast to the Cantorism, logicism and formalism serves not the purpose to make the foundations of the existent mathematics more reliable, but the purpose by means of critical reconsideration of the principles and methods of the existent mathematics on the whole starting from philosophical positions of Descartes and especially from Kant’s philosophy of mathematics to purify it from all unreliable elements of its content.

2) In the course of development of physics at the boundary between 19th and 20th centuries a cardinal reconsideration of various principles of classical physics took place. This reconsideration has resulted into formation of non-classical physical theories as relativity theory and (the initial) quantum theory, containing assertions unacceptable from the point of view of classical physics. A similar transition from classical to corresponding non-classical theories took place at that time also in other fields of the natural sciences.

3) The fundamental change in the foundations of mathematics and natural sciences (in explicit form for the case of mathematics, and in a considerable degree in implicit form for the case of natural sciences), in particular in their principles and methods, which demonstrates a transition from one historical stage of their development – the stage of classical science, to another historical stage – the stage of so called non-classical science, at the boundary between 19th and 20th centuries, naturally was connected with critical attitude to the foundations of old, classical theories, provoking lively interest to some epistemological, ontological and methodological problems and in such a way it reflected into intensification and deepening of interconnection between philosophy and special sciences, on one hand raising new philosophic-methodological problems, and on the other hand modifying the form of their treatments into direction of analytical treatment. Moreover, in the framework of different conceptions concerning foundations of mathematics (cf. fact 15, item “1)”) more or less, in an explicit or at least in an implicit form, corresponding conceptions concerning philosophy of mathematics were proposed as their internal part. As a result of this process of intensification and deepening of interconnection between philosophy and special sciences, just mentioned, an initial phase of the process of formation of philosophy of science in the framework of the concrete scientific investigations began.

For a more concrete information it will be mentioned here: **(1)** George Cantor, Gottlob Frege and Luitzen Egbertus Jan Brouwer set good examples of interrelation of elaboration of new fundamental scientific ideas opening new directions of further development of science with profound treatment of fundamental philosophical problems the results of which namely has played a leading role in the process of concrete scientific investigations; **(2)** Ernst Mach, Ludwig Boltzmann, Henri Poincaré, Leopold Kronecker, Alfred North Whitehead, Bertrand Russell set good examples of great scholars in their special field of science who are in the same time distinguished philosophers elaborating new general philosophical conceptions; **(3)** one may say that at all a lot of the new fundamental scientific ideas formulated at this period of time were connected in one or other way with acceptance of corresponding philosophical views.

So, ultimately, the connection between philosophy on the one hand and mathematics and natural sciences on the other hand at the boundary between 19th and 20th centuries played a decisive, immediate role of preparation of the formation of philosophy of science as a relatively separate field of the scientific thought, which will begin to realize during the next stage of the history of science.

Fact 16.

1) During the first half of 20th century (after “Principia Mathematica”, 1910-1913) essential deepening of the process of modification of the foundations of various sciences (first of all in mathematics, however also in the physics as well as in other branches of the natural sciences) took place realizing the preparatory tendencies which were formed in the initiatory phase of the transition from the classical science to non-classical science at the boundary between 19th and 20th centuries (cf. fact 15 above) and resulting into decisive entering of the human knowledge into the phase of non-classical science. As a consequence of this circumstance a lot of new questions of methodological character about scientific knowledge and research were brought up, the decision of which required reconsideration of various epistemological and ontological problems in order to obtain theoretical basis for substantiation of new methodological approaches to the treatment of various scientific problems. So, generally speaking, the process of development of science during just mentioned period of time has brought up essentially new philosophical problems requiring essentially new way of their treatment, ultimately resulting into formation of the philosophy of science as a separate field of the knowledge.

2) This process of formation of philosophy of science as a separate field of the knowledge proceeded in different degree and form for the case of different sciences. It will follow below a short text by way of presentation of the mentioned above on a more concrete level.

2.1) In the course of very intensive multilateral development of mathematics during the first half of 20th century (after “Principia Mathematica”, 1910-1913) the foundations of mathematics were formed as a particular field of mathematical investigations chiefly as a result of the research work of David Hilbert since 1917. Partly in close connection with the work in the field of foundations of mathematics one may say the logic obtained an intensive development resulting into strengthen its character as mathematical science, i.e. its position as an essential part of mathematics (not as philosophical discipline, as it was regarded traditionally in former times) and in the same time it revealed its potential for foundational studies in science, on the first place in mathematics, but also in the various other fields of scientific knowledge.

2.1.1) Developing further the ideas concerning foundations of geometry, logic and arithmetic which were published in the interval of time 1899-1905 (cf. fact 15, item “1.3”), in a series of articles published in the interval of time 1917-1931 Hilbert elaborated a general conception of *foundations of mathematics in its entirety (as a whole)*, representing the position of the formalism in its completeness in a fundamental two-volumes work composed jointly with Paul Bernays “Grundlagen der Mathematik” (“Foundations of Mathematic”, 1934-1939; cf. [22]), with the audacious ambition to reply in a decisive way to Brouwer’s criticism in address of classical mathematics and in the same time to save mathematics for ever from appearance of any contradictions.

2.1.2) So, during 20-ties and 30-ties years of 20th century a lively discussion on the various problems of foundations of mathematics was carried on between the three directions - logicism, intuitionism and formalism. In this discussion it was revealed in particular a controversy between following three classes of mutually opposite approaches to formulation and solution of foundational problems in mathematics:

2.1.2.1) The first class of mutually opposite approaches to formulation and solution of foundational problems in mathematics: **(1)** contentful (i.e. with regard to the contents) approaches, treating mathematical concepts as possessing specific proper content – logicism and intuitionism (as well as cantorism); and **(2)** formal, treating the mathematical concepts initially as

devoid of any proper content however they obtain definite content by means of postulation of given system of axioms – formalism.

2.1.2.2) The second class of mutually opposite approaches to formulation and solution of foundational problems in mathematics: **(1)** existential approach, treating the mathematical objects as something possessing proper existence – logicism and formalism (it is especially characteristic feature of the cantorism, accepting the very powerful abstraction of actual infinity); and **(2)** processional (in a broad sense) approach, treating the concepts and statements of mathematical theories as result of their engendering in a time-consecution – logicism and more general speaking – mathematical constructivism.

2.1.2.3) The third class of mutually opposite approaches to formulation and solution of foundational problems in mathematics: **(1)** accepting in the foundations of mathematics and in some way *very powerful abstractions* as the concept of actual infiniteness, as well as *very powerful methods* of proof of mathematical theorems, for example connected with application of the law of excluded third (or, of excluded middle: “ x or non- x ”) in the classical logic to infinite sets – logicism and formalism (as well as cantorism) and **(2)** the tendency of *limitations of the means* used in the construction of mathematical theories in particular connected with rejection of the mentioned above abstractions and methods of proof of mathematical theorems, just mentioned above in order to ensure the reliability of the mathematical statements and proofs – intuitionism (such kind of approach to foundantional studies in mathematics is characteristic feature also for the nominalism mentioned below – cf. item “2.1.4”).

2.1.3) Every one of these three directions in the foundations of mathematics mentioned above in one or other way and degree contributed to further development of mathematics in new directions and demonstrated in such a way to be fruitful. Besides in the framework of each of these three directions corresponding philosophy of mathematics – logistic, intuitionistic and formalistic, were developed.

2.1.4) After World War II a new direction in the field of foundations of mathematics which may be characterized as near to the intuitionism was formed – mathematical nominalism, connected with application of requirements of the nominalistic epistemology, which historical root is in the medieval theology, philosophy and logic, to the way of construction of mathematics, on the first place of the set theory.

2.1.5) Parallel to the intensive work on global foundations of mathematics in its integrity during this period of time also intensive foundational

investigations on more concrete level took place, besides in the spirit of early Hilbert's formalism connected with application of abstract axiomatic method. I shall give below three examples in this respect.

2.1.5.1) In 1930-1931 Van der Waerden proposed in his two-volumes work "Moderne Algebra" (in later editions renamed as "Algebra") a systematic modern exposition of the algebra in a very abstract and elegant form elaborated on the bases of lectures of Emmy Noether (1882-1935) and Emil Artin (1898-1962). It begins with the following words: "Die "abstrakte", "formale" oder "axiomatische" Richtung, der die Algebra ihren erneuten Aufschwung verdankt, hat vor allem in der *Gruppentheorie*, der *Körpertheorie*, der *Bewertungstheorie*, der *Idealtheorie*, und der *Theorie der hyperkomplexen Zahlen* zu einer Reihe von neuartigen Begriffsbildungen, zur Einsicht in neue Zusammenhänge und zu weitreichenden Resultaten geführt. In diese ganze Begriffswelt den Leser einzuführen, soll das Hauptziel dieses Buches sein.

Stehen demnach allgemeine Begriffe und Methoden im Vordergrund, so sollen doch auch die Einzelresultate, die zum klassischen Bestand der Algebra gerechnet werden müssen, eine gehörige Berücksichtigung im Rahmen des modernen Aufbaus finden." [23, S. 1].

The first chapter of the book is entitled "Numbers and sets", and its first paragraph – "Sets". So, the set theory is the ultimate basis of construction of the various algebraic structures.

2.1.5.2) Since 1919 Richard von Mises started a series of articles devoted to critical reconsideration of the foundations of probability theory, which was explicitly expressed by means of the title of one of them: "Grundlagen der Wahrscheinlichkeitsrechnung" ("Foundations of probability theory" – cf. [24], [25]) followed up by a monograph "Wahrscheinlichkeit, Statistik und Wahrheit" (1928, in English: "Probability, Statistics and Truth", cf. [26], [26a]).

The central point of this reconsideration of the foundations of probability theory consists in the critical reconsideration of the definition of its basic concept – "probability". Inspired by the philosophy of Ernst Mach, von Mises rejected the classical definition of probability introduced by Laplace and accepted in the textbooks on probability theory at that time as based on a priori idea about "equally likely cases" treated as "equally probable cases" and make attempt to define the concept of probability on the bases of a posteriori concept concerning frequency. Nevertheless that von Mises was a member of the Vienna Circle his treatment of probability theory on the basis of frequency definition of the concept of probability is more in the spirit of Mach's positivism than in the spirit of logical positivism of the Vienna Circle. It is a source of positivistic

treatment of various problems of the theoretical physics at that time, especially of statistical mechanics and quantum mechanics, as well as of general problems of the epistemology, which is demonstrated by means of its incorporation in the systemic exposition of positivistic philosophy in the treatise “Kleines Lehrbuch des Positivismus – Einführung in die empiristische Wissenschaftsauffassung”, cf. especially “IV Abschnitt. Kausalität und Wahrscheinlichkeit” (cf. [27]; “Concise textbook of the Positivism – Introduction into empirical scientific conception”, cf. especially “Chapter IV. Causality and Probability”).

Note to item “2.1.5.2)” of fact 16 in the section 0.2.1. A treatment of the mathematical concept “probability” in the spirit of the logical positivism is proposed by Rudolf Carnap by means of distinction between two types of explication of it – logical and empirical (cf. [28], [29]).

In 1933 A.N. Kolmogoroff published a monograph entitled “Grundbegriffe der Wahrscheinlichkeitsrechnung” (cf. [30], in English: “Fundamental concepts of the Probability Theory”) the purpose of which according to its author is to propose an axiomatic substantiation of the probability theory by means of including its foundations into the series of general concepts of mathematics. In accordance with this purpose Kolmogoroff proposed axiomatic definition of the mathematical concept of probability starting from the abstract concepts of the set theory and on this basis further construction of the next floors of probability theory by means of formal logical deduction. Besides, immediately after formulation of the four axioms concerning the concept of probability Kolmogoroff discussed the problem of attitude of the abstract axiomatic definition of probability to the concrete empirical data. In this connection using some ideas of von Mises without accepting his philosophical position he proposed conceptual (in German: inhaltlich) interpretation of the abstract axiomatic definition of probability by means of empirical concepts of set of observable repeated events and of their frequency. In such a way the probability theory which sometimes had provoked the question whether it is really a mathematical theory or rather it belongs to the natural sciences took shape of mathematical theory par excellence, in the same time however the principle possibility of its application to the empirical investigations of the reality (in particular, of its application to the field of sciences constructed on empirical basis, as theoretical physics for example) is demonstrated.

With publication of Kolmogoroff’s axiomatic system of probability the mathematical foundations of probability theory were established. So, as its historical result the process of formation of probability theory as a mathematical science was completed and further developed.

2.1.5.3) In 1939 a French collective of mathematicians under the pseudonym “Nicolas Bourbaki” began realization of an project for composition

of a monograph in many volumes entitled “*Éléments de Mathématique*” the purpose of which bears a resemblance to one of Euclid’s “*Elements*”: to reconstruct systematically, in uniform axiomatic way almost all current mathematics. The basic difference from Euclid’s approach to such kind of task consists in the choice of basis for such a construction – while the subject of investigations of Euclid’s “*Elements*” as well as in the ancient mathematics at all are figures and numbers, the ultimate basis for introducing various mathematical entities in Bourbaki’s “*Éléments de Mathématique*” is the abstract set theory. The first part of this monograph is entitled “*Les structures fondamentales de l’analyse*” and consists of six books, the first of which is “*Théorie des ensembles*”. In the course of concrete work directed to realization of this project they (I have in mind the members of the collective named “*Nicolas Bourbaki*”) expressed partly explicitly and partly implicitly, their conception concerning foundations of mathematics, in particular their proper treatment of the axiomatic method and of the concept “*mathematical structure*” which plaid a leading role in the process of realization of their project mentioned above.

2.1.6) About the middle of 20th century strengthening of the term “*foundations of mathematics*” took place in the language of mathematics. This fact was manifested in particular by means of publication of the book “*The foundations of mathematics*” by E.W. Beth (1959, second revised edition 1965) in the book-series “*Studies in Logic and the foundations of mathematics*” (of the North-Holland Publishing company, Amsterdam) which can be regarded as an excellent textbook on the subject, combining both its aspects philosophical and logico-mathematical.

2.1.7) During the first half of 20th century after 1913, partly starting from the achievements of Whitehead’s and Russell’s “*Principia Mathematica*”, the symbolic logic obtained rapid many-sided development chiefly by the efforts of mathematicians with philosophical view on their proper problems and philosophers with mathematical style of thinking. Let be mentioned in this connection: **(1*)** formation and fundamental elaboration of the metalogic, **(2*)** construction and elaboration of a variety of non-classical logical systems, **(3*)** attempts for application of logical systems in various fields of scientific knowledge. This development of the symbolic logic created new conditions for foundational studies and in the same time for formation of the philosophy of science.

2.2) In the course of development of physics since the very end of 19th c. and during the first half of 20th c. a cardinal reconsideration of various principles of classical physics took place. This reconsideration has resulted into

formation of new physical theories as quantum theory, quantum mechanics, relativity theory, containing assertions **(1)** unacceptable from the point of view of corresponding classical theories in physics and moreover **(2)** which turned out to be discrepant in respect to our intuition (with which namely the principles of classical physics are compatible). Besides the principle *new content* of the new, non-classical physical theories mentioned above quite naturally is connected with *new form* of its elaboration, involving: **(1*)** application of new, more complicated mathematical theories and methods developed during the second half of 19th c. and the first half of 20th c. as tensor analysis, non-Euclidean geometry, functional analysis, **(2*)** constructing of models instead of theories directly speaking about the studied phenomena (for example Bore's model of the atom of hydrogen), **(3*)** using appropriate logical systems in the treatment of physical problems and even invention of special logical systems (as for example the so called quantum logic) for such purposes. These deep changes in the form of treatment of physical phenomena in the new type of physical theories developed since the end of 19th c. and during first half of 20th c. provoked problems of epistemology, ontology, methodology of scientific knowledge, and because of that almost every physicist who essentially contributed for reconsideration of principles of classical physics, in close relation with treatment of proper problems of physics subjected to examination also the relevant philosophical problems of modern physics.

2.3) During the first half of 20th c., especially because of development of genetics, a serious reconsideration of Darwin's evolution theory took place which also provoked discussion of relevant philosophical problems.

2.4) The process of delimitation between philosophy of science and philosophy of life developed during 19th c., mentioned above in the fact 14, item "4.3)", and more distinctive at the boundary between 19th and 20th centuries, mentioned above in the fact 15, item "3)", is coming into new stage of development during the first half of 20th c., characterized in the most general way by its explication proposed by Richard Müller-Freienfels in "Die Philosophie des zwanzigsten Jahrhunderts in ihren Hauptströmungen" (1923) as follows:

"...die Philosophie der Gegenwart gewiß keine Einheit ist,... sie aber durch den polaren Gegensatz zweier Hauptparteien charakterisiert werden muß, die ich als *Wissenschaftsphilosophie* und *Lebensphilosophie* kennzeichne.

Beide Begriffe sind natürlich Sammelnamen für sehr verschiedene... Richtungen, innerhalb deren jedoch sowohl methodisch als im Ziel unverkennbare Gemeinsamkeiten bestehn. Die Wissenschaftsphilosophie will zunächst eine *Philosophie der Wissenschaft* sein, sie will ausgehen von den

Einzelwissenschaften, in denen sie die festesten Fundamente für Aufdeckung der logischen Struktur der bestehenden Wissenschaften, die festen Gesetze für alles synthetische Denken zu finden glaubt, sie hofft, besonders durch Aufdeckung der logischen Struktur der bestehenden Wissenschaften, die festen Gesetze für alle menschliche Erkenntnis zu gewinnen. Deshalb will sie auch die Philosophie als Wissenschaft betreiben, sie will die strenge Methodik, die in den Einzelwissenschaften zu so wertvollen Ergebnissen geführt hat, auch in der Philosophie beibehalten und glaubt so allein der Philosophie ... innere Festigkeit und Verlässlichkeit geben zu können... Deshalb ist die Methode der meisten Wissenschaftsphilosophen streng rational...

Ganz anders stellt sich die *Lebensphilosophie* dar. Sie nimmt die Wissenschaft nicht ... als den festen Grundstock alles Wissens hin, sondern sieht in ihr... nur eine Form des Lebens, eine dem Leben dienende Konstruktion des Geistes. Sie... ist deshalb zunächst *Philosophie des Lebens*... Philosophie ist ihnen nur eine Form des Lebens, ihr Denken ist Philosophie *als* Leben.

... Es findet sich mannigfache Verbindungen und Zwischenformen zwischen den großen Parteien.” [31], S. 6-7.

This conception about delimitation in the philosophical thought of 19th c. between two types of philosophies – philosophy of science and philosophy of life, is expressed distinctively by means of the way of presentation of the philosophical thought during 19th c. by Müller-Freienfels – he expounded the last one in following two parts of the book:

“Erster Teil. Philosophie der Wissenschaft und Philosophie als Wissenschaft.”

“Zweiter Teil. Philosophie des Lebens und Philosophie als Leben.”

This explicit description in the most general way of the implicit, however outstanding distinction between philosophy of science and philosophy of life during 19th c., published in 1923, in the same time actually turned out to serve as prescription of the tendency which had to follow and further to develop the philosophical thought during 20th c.

The various directions in the framework of the philosophy of science realized during 20th c. actually manifested in one or other way specifications of its concept in a more concrete level. Here I shall limit myself only to mention that soon after publication of Müller-Freienfels's book Rudolf Carnap in his first fundamental monograph “Der logische Aufbau der Welt” (1928) stressed that the transition from the traditional philosophy occupied by metaphysics to scientific philosophy, or to philosophy as a science, free from any metaphysics, is a result of formation during the recent decades of a new logic, namely of the symbolic logic, and its application as a technical tool to treatment of

philosophical problems, or speaking more precisely as a technical tool for clarification of the philosophical concepts and for purification of philosophical problems:

“Die Mathematiker haben in den letzten Jahrzehnten eine neue Logik aufgebaut. Sie sind durch die Not, durch die Grundlagenkrise der Mathematik dazu gezwungen worden, da die alte Logik in dieser Krise vollständig versagte...

Es ist historisch verständlich, daß die neue Logik zunächst nur im engeren Fachkreise der Mathematiker und Logiker Beachtung gefunden hat. Ihre hervorragende Bedeutung für die gesamte Philosophie wird nur von wenigen geahnt; ihre Auswertung auf diesem weiteren Felde hat kaum erst begonnen. Wenn die Philosophie willens ist, den Weg der Wissenschaft (im strengen Sinne) zu betreten, so wird sie auf dieses durchgreifend wirksame Mittel zur Klärung der Begriffe und zur Säuberung der Problemsituationen nicht verzichten können.” [32], S. XVII-XVIII.

In his excellent article “Überwindung der Metaphysik durch logische Analyse der Sprache” Carnap essentially developed further his conception about distinction of the philosophy of science from the philosophy of life and from the old metaphysic (cf. [33], [33a]).

In his next fundamental monograph “Logische Syntax der Sprache” (1934) Carnap reinforced and concretised further this thesis accepting the logical syntax of the language of science to be the logic of science and namely the last one replaced the traditional philosophy, i.e. according to Carnap:

Scientific philosophy, respectively philosophy of science =
= logic of science = logical syntax of the language of science.

“...Die logische Syntax will nun ein Begriffsgebäude, eine Sprache liefern, mit deren Hilfe die Ergebnisse logischer Analyse exact formulierbar sind. Philosophie wird durch Wissenschaftslogik, d.h. logische Analyse der Begriffe und Sätze der Wissenschaft ersetzt; Wissenschaftslogik ist nichts anderes als logische Syntax der Wissenschaftssprache.” [34], S. III-IV.

2.5) A characteristic feature of development of scientific knowledge during the first half of 20th century was the following one: raising and treating peculiar philosophic-methodological problems of various fields of science in close connection with the research of their proper problems in natural way grew into raising and treating corresponding general philosophic-methodological problems in their abstract form besides the subsequent elaboration of philosophic-methodological conceptions of great generality afterwards served as starting points to come down from generalities to particulars, demonstrating in such a way the close connection between general and peculiar in the process of

scientific research. So, such kind of transitions in the course of scientific investigations was resulting into outstanding *distinction* and in the same time into *close connection* between **(1) general philosophy of science**, treating general philosophical problems of science, on the one hand, and **(2) special (particular) philosophies of science**, treating specific philosophic problems of corresponding special sciences (respectively, corresponding fields or branches of the science), on the other hand.

2.6) During the first half of 20th century the interdisciplinary investigations in the scientific knowledge were reinforced and obtained new forms.

2.6.1) In the course of revealing **(1)** interconnections and interactions between different phenomena studied by contiguous sciences, or **(2)** different aspects of one and the same phenomenon, besides they turned out to be relevant in an alternative way to two different contiguous sciences, or **(3)** that some phenomena studied in a given science can be deduced from or merely explained by means of phenomena studied in another science, so that the second kind of phenomena turned out to be more fundamental with respect to first kind of phenomena and in such a way the science of the second kind of phenomena to play a role of theoretical foundation of the science of first kind of phenomena **(1→3)** interdisciplinary scope of investigations on the boundary field between these two contiguous sciences, as for example between physics and chemistry, or between chemistry and biology, took more and more a place, resulting into further development or initial origination of interdisciplinary scientific theories as physical chemistry, biochemistry and so on.

2.6.2) Owing to universal character of the general philosophy of science and in particular of the general methodology of science in respect to various branches of the science the treatment of their problems required to make their discussion by the joined forces of specialists from different fields of scientific knowledge, otherwise speaking it required application in some broader sense of interdisciplinary approach. Indeed, during this period of time mentioned above two famous interdisciplinary groups of scholars for discussion of actual general methodological problems (and in close connection with them also of epistemological and ontological ones) were established: **(1)** Vienna circle, organised and led by Moritz Schlick (1882-1936) – a philosopher, well-educated in physics, and embracing as participants a number of scholars from different branches of the scientific knowledge, and **(2)** seminar on scientific method at the Harvard Medical School in Cambridge, Massachusetts, organised and led by Arturo Rosenblueth – a physician and medical scientist from the same School, and embracing as participants mostly young scientists from the same School but also scholars from different branches of science and from other

universities. Both these forums contributed in their way to development of the interdisciplinary approach in the scientific knowledge.

2.6.2.1) In Vienna circle, side by side with elaborating the logic and semantic approach to the language of science, which often is assumed as the most characteristic feature of the logical positivism, another very important and perspective from my point of view methodological conception was elaborated too – I have in mind the conception of encyclopedism, proposed by Otto Neurath and developed together with Rudolf Carnap, Charles Morris, Niels Bohr, Bertrand Russell, John Dewey, etc. (cf. [35]).

One can treat encyclopedism as synthesis of two fundamental methodological ideas:

2.6.2.1.1) First of all encyclopedism is a way or a form of development of the interdisciplinary approach to scientific knowledge treated as an approach to “unified science” which has to be accomplished in the form of Encyclopedia in many volumes as a result of cooperative activity of scientists of various fields of scientific knowledge and to represent a model of science at the given age.

2.6.2.1.2) On the second place the encyclopedism turned out to be a way to introduce the concept “foundations of science” as a further step of generalisation after formation of the concept “foundations of mathematics”. Speaking on more concrete level this introduction is realised by means of the coinage of the term “Foundations of the Unity of Science” used as title of the two-volume introductory methodological treatise “Towards an International Encyclopedia of Unified Science” (cf. [35]).

2.6.2.2) In the course of activity of the interdisciplinary methodological seminar at Harvard Medical School mentioned above a next, decisive step of the historical process of development of interdisciplinary investigations was made – its founder and leader Arturo Rosenblueth established near connection for joint investigation with the outstanding professor in mathematics in the Massachusetts Institute of Technology, distinguished by broad interests and creative work in various branches of mathematics, including in mathematical logic, as well as in various fields of its application, Norbert Wiener. Besides both had shared one and the same conviction that **(1)** the most fruitful areas for development of the sciences are “those which had been neglected as a no-man’s land between the various established fields” ([36], p. 2) and **(2)** “a proper exploration of these blank spaces on the map of science could only be made by a team of scientists, each a specialist in his own field but each possessing a thoroughly sound and trained acquaintance with the fields of his neighbours” ([36], p.3). So, step by step they have created a new science treated by them as

an interdisciplinary field of scientific knowledge between biology and technology, the fundamental ideas of which are exposed in the Norbert Wiener's book "Cybernetics: or Control and Communication in the Animal and the Machine" (1948, cf. [36]).

Fact 17.

1) By way of the next years after publication of Norbert Wiener's book "Cybernetics" (1948) it turned out that the conceptions formulated in it are applicable also to various other fields of science immediately demonstrating in such a way that cybernetics is an interdisciplinary field of science of a new type, principally different from such kind of interdisciplinary fields of sciences on the boundary between two fundamental sciences as physical chemistry, which is elaborated on the boundary between physics and chemistry, or biochemistry, which is elaborated on the boundary between chemistry and biology. Subsequently various other interdisciplinary theories have been proposed of such a new type, which are simultaneously applicable to various fields of science. This is *a new interdisciplinary tendency in the history of science originating since 1948 with publication of Norbert Wiener's "Cybernetics"*.

2) *Tendency of mathematization of scientific knowledge*: Mathematics, which during 17th and 18th centuries was developed exclusively in connection with the problems and requirements of formation of the physical sciences and the needs of technology, during 19th and first half of 20th century step by step extended the sphere of its influence to various fields of scientific knowledge and during second half of 20th century, enriched with new branches, in particular with various branches of the so-called "qualitative mathematics" like various systems of mathematical (symbolic) logic and of abstract algebra, obtained the form of universally applicable science.

3) *Tendency of computerization of scientific knowledge*: The efforts since the time of Raimundus Lullus to invent mechanism realizing mental processes (or, more precisely speaking: model of mental processes) about during the middle of 20th century was crowned with success and during second half of 20th century computers obtained various applications, they became a powerful, many-functional implement of the intellectual work, playing considerable role in the process of scientific research.

4) *Tendency of modellization of scientific knowledge*: Some forms of modelling were well known in antiquity as a factor for production; in the course of the history of science modelling in its various forms obtained important role to an increasing degree and during second half of the 20th century it became a fundamental, many-functional tool of scientific knowledge. In particular at that

historical period of development of science the mathematization and the computerization of scientific knowledge mentioned above in items “2)” and “3)” to an considerable degree obtained the form of mathematical and computer modelling.

Note to fact 17. As a result of rapid growth of the scientific knowledge during second half of 20th century naturally also other events relevant to the question about formation and modification of the characteristic features of the science took place in it. Here we have to limit ourselves to notice only these four tendencies of very general character and with deep roots in the history of science which are mentioned above because the so called above other events do not belong to the history of science, but they are the very subject of our actual contemporary investigation.

0.2.2. Explication of the basic concepts included in the expression of order in the general formulation of task of present investigation

0.2.2.1. Explication of the concept “science”

0.2.2.1.0. Preliminary notes concerning the way of explication of the concept “science”

We shall introduce here two types of explications of concept “science”, conditionally named as it follows:

1) *Formal explications*, possessing essentially more general character denoting corresponding classes of possible more concrete explications, besides these classes as well as the corresponding explications merely fix various aspects of the science, so in such a sense they obtain formal character and significance; and

2) *Contentful explications* (i.e. explications with regard to the contents), containing expressly concrete, conceptual character.

Note to section 0.2.2.1.0. These two types of explication of concept “science” can be expressed extensionally by means of two different types of dynamic sets (respectively, of different types of conceptual vectors and matrixes):

1) the formal explications of concept “science” can be represented by means of conjunctive set (respectively, of conjunctive conceptual vector or matrix), i.e. by means of set all elements of which are jointly acceptable (respectively, by means of conceptual vector or matrix all components of which are jointly acceptable), and

2) the contentful explications of concept “science” can be represented by means of strictly disjunctive set (respectively, by means of strictly disjunctive conceptual vector or matrix), i.e. by means of set the elements of which are alternatively acceptable, so that if a given element from the set is accepted, then all other elements from it are rejected (respectively, by means of conceptual vector or matrix the components of which are alternatively acceptable, so that if a given component of a given conceptual vector or matrix is accepted, then all other components of the same are rejected).

The conceptual sets, vectors, matrixes **A** which are conjunctive will be denoted by means of the sign “&” as “&A”; those which are strictly disjunctive - by means of the sign “V” as “VA”.

0.2.2.1.0.ω. Corollary from the preliminary notes concerning the way of explication of the concept “science”

Taking into account what was mentioned in the preliminary notes concerning the way of explication of concept “science” in the present section 0.2.2.1.0, in order to achieve appropriate (with respect to the task of present investigation) explication of the concept “science”, we have:

1) first of all taking into account the set of basic facts of history of science established in section 0.2.1 to construct a conjunctive set of formal explications of the concept “science” revealing in a systematic way various aspects of the science;

2) on second place, to make a choice of such formal explication or explications of the concept “science” which is or which are relevant to the task of present investigation;

3) on third place, on the empirical basis of the set of basic facts of history of science revealed in section 0.2.1 and taking into account some considerations of general character to formulate and substantiate a contentful explication (i.e. an explication with regard to the contents) of the concept “science”.

0.2.2.1.1. Conjunctive sets of formal explications of the concept “science” revealing in a systematic way various aspects of the science

0.2.2.1.1.1. First step of the process of systematic revealing various aspects of the science: construction of concept-vectors $\& [Sc^i]_1$ and $\& [Sc_j]_1$, and of concept-matrix $\& [Sc_j^i]_{1,1}$ of possible formal explications of the concept "science"

On the first step of process of a semantic analysis of the sense of term "science", i.e. of the content of concept "science", denoted symbolically as “Sc”, taking into account the set of basic facts of history of science established in section 0.2.1, we shall distinguish between two basic types of their conjunctive explications, corresponding to two qualitative parameters, everyone of which obtains as possible values some concepts correspondingly playing the role of possible schemes of explications of the concept “science”:

1) First type of explications, corresponding to the parameter **i**, involves two possible qualitative concept-values “corpus (**crp**) - process (**prc**)”, which in the

considerations following below play the role of predicates. We shall express this type of explications by means of a conjunctive 2-column-concept-vector:

$$\&[\mathbf{Sc}^i]_1 = \begin{bmatrix} \mathbf{Sc}^{crp} \\ \mathbf{Sc}^{prc} \end{bmatrix}, \quad \{1\}$$

where the symbols “ $\mathbf{Sc}^{crp}$ ” and “ $\mathbf{Sc}^{prc}$ ” obtain the following two kinds of very abstract interpretations, or, rather, the following two kinds of schemes of interpretations:

- 1.1) $\mathbf{Sc}^{crp}$ - science as a corpus,
- 1.2) $\mathbf{Sc}^{prc}$ - science as a process.

2) Second type of explications, corresponding to the parameter **j**, involves two possible qualitative concept-values “knowledge plus immediate factors for it (**knl+ifctr**) - social phenomenon (**sclphn**)”, which in the considerations following below play the role of individuals. We shall express this type of explications by means of a conjunctive 2-row-concept-vector

$$\&[\mathbf{Sc}_j]_1 = \&[\mathbf{Sc}_{knl+ifctr} \quad \mathbf{Sc}_{sclphn}], \quad \{2\}$$

where the symbols “ $\mathbf{Sc}_{knl+ifctr}$ ” and “ $\mathbf{Sc}_{sclphn}$ ” obtain the following two kinds of very abstract interpretations, or, rather, the following two kinds of schemes of interpretations:

2.1) $\mathbf{Sc}_{knl+ifctr}$ - science as a form of hard knowledge together with the immediate factors of its production, i.e. treatment of the science from the point of view of its aspect expressing its proper, inner content as an essential form of knowledge involving also the immediate factors of its generation;

2.2) $\mathbf{Sc}_{sclphn}$ - science as a form of social phenomenon, i.e. treatment of the science from the point of view of its social aspect expressing the mutual connection of the form of knowledge mentioned above (cf. item “2.1”) with the various kinds of social entities and events.

3) These two classifications of the explications of concept “science” corresponding to the parameters **i** and **j** as well as these four types (respectively, schemes) of explications of concept “science” involved in them $\mathbf{Sc}^{crp}$, $\mathbf{Sc}^{prc}$, $\mathbf{Sc}_{knl+ifctr}$ and $\mathbf{Sc}_{sclphn}$, treated in items “1)” and “2)”, as it was already mentioned above, are too abstract, i.e. they are too much devoid of

specific content, so therefore they are not able to serve as effective characteristics of various aspects of science. In the same time, however, owing namely to their very general form, correlated with their very high degree of abstractedness, they may serve as initial constructive elements for a process of systematic construction of various possible explications of the content of intuitive concept "science", so that in the course of such a process they turn out to become step by step ever richer and richer in content.

4) We shall set all possible combinations of components taken from the two parameters just mentioned above in the items “1)” and “2)” in a systematic way treated as a result of giving effect to the operation “Cartesian product” upon the corresponding 2-concepts-vectors ${}_&[\mathbf{Sc}^i]_1$ and ${}_&[\mathbf{Sc}_j]_1$ represented by means of the equations {1} and {2} :

$${}_&[\mathbf{Sc}_j^i]_{1,1} = ({}_&[\mathbf{Sc}^i]_1 \otimes {}_&[\mathbf{Sc}_j]_1). \quad \{3\}$$

Writing the same in a more detailed form, we obtain:

$${}_&[\mathbf{Sc}_j^i]_{1,1} = \begin{bmatrix} \mathbf{Sc}^{crp} \\ \mathbf{Sc}^{prc} \end{bmatrix} \otimes {}_&[\mathbf{Sc}_{knl+ifctr} \mathbf{Sc}_{sclphn}] = \begin{bmatrix} \mathbf{Sc}_{knl+ifctr}^{crp} & \mathbf{Sc}_{sclphn}^{crp} \\ \mathbf{Sc}_{knl+ifctr}^{prc} & \mathbf{Sc}_{sclphn}^{prc} \end{bmatrix}. \quad \{4\}$$

2) Taking into account (1) both schemes of interpretation of the components of concept-vectors ${}_&[\mathbf{Sc}^i]_1$ and ${}_&[\mathbf{Sc}_j]_1$ above, as well as (2) the set of basic facts of history of science established in section 0.2.1, (1→2) we can propose here the following interpretation of the components of matrix ${}_&[\mathbf{Sc}_j^i]_{1,1}$:

2.1) The denotation “ $\mathbf{Sc}_{knl+ifctr}^{crp}$ ” means "science as a corpus of knowledge together with the immediate factors of scientific work". It may be further divided into following two more concrete concepts of science:

2.1.1) The concept of science treated as form of knowledge distinguished itself by relative stability for a historical interval of time – it will be expressed by the use of the word “hard” and will be denoted by “ $\mathbf{Sc}_{hrdknl}^{crp}$ ” (to be read as “science as a corpus of hard knowledge”). We have further to take also into account that this form of knowledge and in the same time of the treatment of science as a form of knowledge is essentially divided into following three fundamental forms:

2.1.1.1) the science as a corpus of hard empirical knowledge, which will be denoted by “ $\mathbf{Sc}_{heknl}^{crp}$ ”;

2.1.1.2) the science as a corpus of hard theoretical knowledge, which will be denoted by “ Sc_{htknl}^{crp} ”; and

2.1.1.3) the science as a corpus of hard pragmatic knowledge (or, otherwise speaking: as scientific foundations of hard pragmatic knowledge as technologies agrotechnics, medicine), which will be denoted by “ Sc_{hpknl}^{crp} ”.

2.1.2) The concept of science treated as a set of immediate factors of scientific work, regarded in their aspect to be immediately involved in the information processing in the course of scientific research resulted into production of hard scientific knowledge, which will be denoted by “ $\text{Sc}_{knlifctr}^{crp}$ ” (to be read as “science as a corpus of hard knowledge immediate factors”), involves:

2.1.2.1) a set of scientists as immediate subjects of scientific work, denoted by “ Sc_{knlsbj}^{crp} ” (to be read as “science as a corpus of knowledge subjects”), and

2.1.2.2) a set of means of scientific work as measuring instruments, observational and experimental devices, electronic devices for information and word processing, means for record of the results of scientific research and the like - it will be denoted by “ Sc_{knlmns}^{crp} ” (to be read as “science as a corpus of knowledge means”).

2.1.ω) So, the explication of the concept “science” in the sense denoted by “ $\text{Sc}_{knl+ifctr}^{crp}$ ” can be represented in a large scale as a 2-row-concept-vector in the following way:

$$\& [\text{Sc}_{knl+ifctr}^{crp}]_1 = \& [\text{Sc}_{hrdknl}^{crp} \quad \text{Sc}_{knlifctr}^{crp}]. \quad \{5\}$$

In view of the mentioned above in item 2.1.1 the explication of the concept “science” denoted by “ Sc_{hrdknl}^{crp} ” can be represented as a 3-row-concept-vector in the following way:

$$\& [\text{Sc}_{hrdknl}^{crp}]_1 = \& [\text{Sc}_{heknl}^{crp} \quad \text{Sc}_{htknl}^{crp} \quad \text{Sc}_{hpknl}^{crp}]; \quad \{6\}$$

as well as in a similar way taking into account what was mentioned above in item “2.1.2)” the explication of the concept “science” denoted by “ $\text{Sc}_{knlifctr}^{crp}$ ” can be represented as a 2-row-concept-vector in the following way:

$$\& [\text{Sc}_{knlifctr}^{crp}]_1 = \& [\text{Sc}_{knlsbj}^{crp} \quad \text{Sc}_{knlmns}^{crp}]. \quad \{7\}$$

As a result of replacement of $\text{Sc}_{knlifctr}^{crp}$ in the formula {5} by the right site of the equation {7} we obtain the following more detailed (in respect to the formula

{5}) form of concept-vector-representation of the various possible formal explications of the concept “science” denoted by “ $\mathbf{Sc}_{knl+ifctr}^{crp}$ ”:

$$\& [\mathbf{Sc}_{knl+ifctr}^{crp}]_2 = \& [\mathbf{Sc}_{hrdknl}^{crp} \mathbf{Sc}_{knlshj}^{crp} \mathbf{Sc}_{knlmns}^{crp}]. \quad \{8\}$$

Farther, in result of replacement of $\mathbf{Sc}_{hrdknl}^{crp}$ in the formula {8} by the right site of the equation {6} we obtain the following still more detailed (in respect to the formula {8}) form of concept-vector-representation of the possible formal explications of the concept “science” denoted by “ $\mathbf{Sc}_{knl+ifctr}^{crp}$ ”:

$$\& [\mathbf{Sc}_{knl+ifctr}^{crp}]_3 = \& [\mathbf{Sc}_{hekn}^{crp} \mathbf{Sc}_{htknl}^{crp} \mathbf{Sc}_{hpkn}^{crp} \mathbf{Sc}_{knlshj}^{crp} \mathbf{Sc}_{knlmns}^{crp}]. \quad \{9\}$$

2.2) The denotation “ $\mathbf{Sc}_{sciphn}^{crp}$ ” means "science as a corpus of social phenomena". It involves treatment of the science in the following three forms:

2.2.1) as a set of social aspects (in the given case, respectively, of social elements) of the scientific knowledge in a very broad sense - scientific theories, conceptions, laws, etc., expressing its mutual connection with social entities and events - it will be denoted by “ $\mathbf{Sc}_{sclnkl}^{crp}$ ”, and will be read literally as “science as a corpus of social aspects of scientific knowledge”, however it will be interpreted more precisely speaking in the following way: “scientific knowledge in the very broad sense consisting of social theoretical (or semitheoretical) constructions”;

2.2.2) as a set of separate scientists being members of society at all and more concrete speaking of various social groups as individual persons and so treated in their aspect in the course of their scientific work to be involved in a net of various social relations - it will be denoted by “ $\mathbf{Sc}_{sclind}^{crp}$ ”, and will be read as “science as a corpus of individuals in their social aspect”;

2.2.3) as a set of social groups of scientists as research groups, public scientific organizations, scientific institutions (Academies, Universities, Institutes, laboratories and so on), scientific schools, respectively, movements, etc., forming **(1)** specific social relations between scientists as members of one and the same group, **(2)** social relations between different scientific groups, and **(3)** various kinds of connections of the scientific groups with the society as a whole - it will be denoted by “ $\mathbf{Sc}_{sclgrp}^{crp}$ ” and will be read as “science as a corpus of social groups”, and

2.2.4) as the corresponding equipment like buildings, their furniture, etc., which connect scientists in the course of production of scientific knowledge,

besides treated just in the aspect of their social role - it will be denoted by “ $\text{Sc}_{scl\text{eqp}}^{crp}$ ” and will be read as “science as a corpus of the scientific equipment in its social aspect”.

2.2.ω) So, we can present the formal explication of the concept “science” in the sense denoted by “ $\text{Sc}_{scl\text{phn}}^{crp}$ ” as a 4-row-concept-vector in the following way:

$$\& [\text{Sc}_{scl\text{phn}}^{crp}]_1 = \& [\text{Sc}_{scl\text{knl}}^{crp} \text{ Sc}_{scl\text{ind}}^{crp} \text{ Sc}_{scl\text{grp}}^{crp} \text{ Sc}_{scl\text{eqp}}^{crp}]. \quad \{10\}$$

2.3) The denotation “ $\text{Sc}_{knl+ifctr}^{prc}$ ” means “science as a process of functioning of the corpus of scientific knowledge together with of the immediate factors for its production $\text{Sc}_{knl+ifctr}^{crp}$ ”, besides the word “functioning” is used in the phrase enclosed in inverted comas in a very general sense, which will be subjected to elucidation below.

By analogy with the formulae {5}, {8} and {9} in item 2.1.ω, here we must have the following formulae {11} - {13}, respectively:

$$\& [\text{Sc}_{knl+ifctr}^{prc}]_1 = \& [\text{Sc}_{hrd\text{knl}}^{prc} \text{ Sc}_{knlifctr}^{prc}], \quad \{11\}$$

$$\& [\text{Sc}_{knl+ifctr}^{prc}]_2 = \& [\text{Sc}_{hrd\text{knl}}^{prc} \text{ Sc}_{knl\text{sbj}}^{prc} \text{ Sc}_{knl\text{mns}}^{prc}], \quad \{12\}$$

$$\& [\text{Sc}_{knl+ifctr}^{prc}]_3 = \& [\text{Sc}_{heknl}^{prc} \text{ Sc}_{htknl}^{prc} \text{ Sc}_{hpknl}^{prc} \text{ Sc}_{knl\text{sbj}}^{prc} \text{ Sc}_{knl\text{mns}}^{prc}], \quad \{13\}$$

where the sign “ Sc_x^{prc} ” means “science as a process of functioning of Sc_x^{crp} ”, x is element of the set of all lower indexes of the components of concept-vectors in the right site of equations {11}-{13}.

Now we shall make a step towards clarification of the sense of term “functioning”, respectively “process of functioning”, applied to the phrase “the corpus of scientific knowledge together with the immediate factors for its production” in the first sentence of present item “2.3)”, by means of revealing its more concrete kinds. So, we shall distinguish here between following two kinds of the concept “functioning” as it is applied to the phrase mentioned above:

2.3.1) as process of functioning of the corpus of scientific knowledge together with of the immediate factors for its production $\text{Sc}_{knl+ifctr}^{crp}$ treated *on microlevel* - this is a process of immediate production of scientific knowledge in a given kind as successive posing and solution of scientific problems in the course of scientific investigations in a very limited (from the historical point of view) interval of time - it will be denoted by “ $\text{Sc}_{knl+ifctr}^{miprc}$ ” and on its part it can be

represented here as 2-, 3- and 5-row-concept-vectors by means of equations which are particular cases of {11}-{13}, respectively, obtained from them as a result of replacement of the upper index in them “**prc**” by means of the upper index “**miprc**” as it follows:

$$\& [\text{Sc}_{\text{kn}+\text{ifctr}}^{\text{miprc}}]_1 = \& [\text{Sc}_{\text{hrdknl}}^{\text{miprc}} \text{Sc}_{\text{kn}+\text{ifctr}}^{\text{miprc}}], \quad \{14\}$$

$$\& [\text{Sc}_{\text{kn}+\text{ifctr}}^{\text{miprc}}]_2 = \& [\text{Sc}_{\text{hrdknl}}^{\text{miprc}} \text{Sc}_{\text{knlsbj}}^{\text{miprc}} \text{Sc}_{\text{kn}+\text{ifctr}}^{\text{miprc}}], \quad \{15\}$$

$$\& [\text{Sc}_{\text{kn}+\text{ifctr}}^{\text{miprc}}]_3 = \& [\text{Sc}_{\text{hekn}+\text{ifctr}}^{\text{miprc}} \text{Sc}_{\text{htkn}+\text{ifctr}}^{\text{miprc}} \text{Sc}_{\text{hpknl}+\text{ifctr}}^{\text{miprc}} \text{Sc}_{\text{knlsbj}+\text{ifctr}}^{\text{miprc}} \text{Sc}_{\text{kn}+\text{ifctr}}^{\text{miprc}}]. \quad \{16\}$$

The sign $\text{Sc}_x^{\text{miprc}}$, where the component x is element of the set of all lower indexes of the components of concept-vectors in the right site of equations {14}-{16}, generally speaking means the process of functioning on microlevel of the aspect of science denoted by “ Sc_x^{crp} ” (or, expressing the same in another way, it means the participation of the aspect of science Sc_x^{crp} in the process of posing and solution of problems) in the course of given scientific investigation aimed at producing of new scientific knowledge as a new scientific law, a new scientific theory, etc.;

2.3.2) as process of functioning of the corpus of scientific knowledge together with of the immediate factors for its production $\text{Sc}_{\text{kn}+\text{ifctr}}^{\text{crp}}$ on macrolevel - this is a process of historical development of that aspect of the science which is denoted by “ $\text{Sc}_{\text{kn}+\text{ifctr}}^{\text{crp}}$ ” taking its usual course in a large scale of time - it will be denoted by $\text{Sc}_{\text{kn}+\text{ifctr}}^{\text{maprc}}$ and on its part it can be represented here as 2-, 3- and 5-row-concept-vectors by means of equations which are corresponding particular cases of {11} - {13}, respectively, obtained as a result of replacement of the upper index in them “**prc**” by means of the upper index “**maprc**” as it follows:

$$\& [\text{Sc}_{\text{kn}+\text{ifctr}}^{\text{maprc}}]_1 = \& [\text{Sc}_{\text{hrdknl}}^{\text{maprc}} \text{Sc}_{\text{kn}+\text{ifctr}}^{\text{maprc}}], \quad \{17\}$$

$$\& [\text{Sc}_{\text{kn}+\text{ifctr}}^{\text{maprc}}]_2 = \& [\text{Sc}_{\text{hrdknl}}^{\text{maprc}} \text{Sc}_{\text{knlsbj}}^{\text{maprc}} \text{Sc}_{\text{kn}+\text{ifctr}}^{\text{maprc}}], \quad \{18\}$$

$$\& [\text{Sc}_{\text{kn}+\text{ifctr}}^{\text{maprc}}]_3 = \& [\text{Sc}_{\text{hekn}+\text{ifctr}}^{\text{maprc}} \text{Sc}_{\text{htkn}+\text{ifctr}}^{\text{maprc}} \text{Sc}_{\text{hpknl}+\text{ifctr}}^{\text{maprc}} \text{Sc}_{\text{knlsbj}+\text{ifctr}}^{\text{maprc}} \text{Sc}_{\text{kn}+\text{ifctr}}^{\text{maprc}}], \quad \{19\}$$

The interpretation of components of the conjunctive concept-vectors in the right site of equations {17}-{19} generally speaking consists in process of functioning on macrolevel of the corresponding aspect of science denoted by

“ $\text{Sc}_{\text{knl}+\text{ifctr}}^{\text{crp}}$ ”, or expressing the same in other way, it consists in the historical process of origination and further development of the corresponding $\text{Sc}_{\text{knl}+\text{ifctr}}^{\text{crp}}$.

2.4) By analogy with the case considered in item “2.3)” the denotation “ $\text{Sc}_{\text{sclphn}}^{\text{prc}}$ ” means “science as a process of functioning of the corpus of social phenomena $\text{Sc}_{\text{sclphn}}^{\text{crp}}$ in the course of scientific research and development”. This process is interpreted here in such a general way which allows revealing following two kinds of it:

2.4.1) Science as microprocess of functioning of $\text{Sc}_{\text{sclphn}}^{\text{crp}}$, denoted by “ $\text{Sc}_{\text{sclphn}}^{\text{miprc}}$ ”. In this case the science is treated as a form of process of social actions and of entering into social relations of $\text{Sc}_{\text{sclphn}}^{\text{crp}}$ in the course of scientific research *in a very limited (from an historical point of view) interval of time*. In accordance with the four more concrete explications of the concept science in the sense of $\text{Sc}_{\text{sclphn}}^{\text{crp}}$ mentioned in item “2.2)”, the type of explication of concept science $\text{Sc}_{\text{sclphn}}^{\text{miprc}}$ involves following four still more concrete kinds of its explication:

2.4.1.1) $\text{Sc}_{\text{sclknl}}^{\text{miprc}}$ - science as a process of immediate producing and action of “scientific knowledge in the very broad sense consisting of social theoretical (or semitheoretical) constructions” (cf. item “2.2.1)”) in the course of the intellectual work of individuals.

2.4.1.2) $\text{Sc}_{\text{sclind}}^{\text{miprc}}$ - science as a process of social acting of the separate scientists in the course of their scientific investigations in which they enter into personal interconnections between themselves as well as into various relations of social character with other spheres (i.e. outside of the science) of the society as family, national traditions, political events, dominated world outlook, adopted moral norms, way of life, economic interconnections, etc.;

2.4.1.3) $\text{Sc}_{\text{sclgrp}}^{\text{miprc}}$ - science as a process of social acting of the social groups of scientists as research groups, public scientific organizations, scientific institutions (Academies, Universities, Institutes, laboratories and so on), scientific schools, respectively, movements, etc. forming in such a process of **(1)** specific social relations between scientists as members of one and the same group, **(2)** various kinds of relationship between different scientific groups, and **(3)** various kinds of connections of the scientific groups with the society as a whole;

2.4.1.4) Sc_{scleqp}^{miprc} - science as a process of generation of social relations between the means of scientific work as well as of the other its equipment, on the one hand, and the social entities and events of various forms, on the other hand.

2.4.2) Science as macroprocess of the functioning of Sc_{sclphn}^{crp} , denoted by “ Sc_{sclphn}^{maproc} ” and consisting in a process of historical evolution (i.e. as a process of formation, functioning and further development, or of decay) of that aspect of the science which is regarded as a corpus of social phenomena Sc_{sclphn}^{crp} , taking its usual course *in a large scale of time*. In accordance with the four more concrete explications of the concept science in the sense of Sc_{sclphn}^{crp} mentioned in item “2.2)”, the type of explication of concept science Sc_{sclphn}^{maproc} involves following four still more concrete kinds of its explication:

2.4.2.1) Sc_{sclknl}^{maproc} - science as a historical process of formation, functioning and subsequent development, or of decay of “scientific knowledge in the very broad sense consisting of social theoretical (or semitheoretical) constructions” (cf. item “2.2.1)”), i.e. of that aspect of science which is regarded as a corpus of social phenomena Sc_{sclphn}^{crp} involving various social theories, conceptions, laws, etc.

2.4.2.2) Sc_{sclind}^{maproc} - science as a historical process of initial formation and of subsequent change of the type of scientist as a type of person being chief immediate and atomic subject of the scientific research;

2.4.2.3) Sc_{sclgrp}^{maproc} - science as a historical process of initial formation and of subsequent development (or of decay) of research groups, public scientific organizations, scientific institutions (Academies, Universities, Institutes, laboratories and so on), scientific schools, respectively, movements, etc. in respect of their social characteristics and connections;

2.4.2.4) Sc_{scleqp}^{maproc} - science as a historical process of creation and development of the means of scientific work and of other its equipment in the social context.

2.4.ω) Correspondingly to the formula {10} in item “2.2.ω)”, here we have formulae {20}-{22}, formally obtained from it by means of replacement of the upper index in it “crp**” by means of the upper indexes “**prc**”, “**miprc**” and “**maprc**” for the formulae {20}, {21} and {22}, respectively:**

$$\& [Sc_{sclphn}^{prc}]_1 = \& [Sc_{sclknl}^{prc} \quad Sc_{sclind}^{prc} \quad Sc_{sclgrp}^{prc} \quad Sc_{scleqp}^{prc}] , \quad \{20\}$$

$$\& [\mathbf{Sc}_{sclphn}^{miprc}]_1 = \& [\mathbf{Sc}_{sclknl}^{miprc} \mathbf{Sc}_{sclind}^{miprc} \mathbf{Sc}_{sclgrp}^{miprc} \mathbf{Sc}_{sclqp}^{miprc}], \quad \{21\}$$

$$\& [\mathbf{Sc}_{sclphn}^{maprc}]_1 = \& [\mathbf{Sc}_{sclknl}^{maprc} \mathbf{Sc}_{sclind}^{maprc} \mathbf{Sc}_{sclgrp}^{maprc} \mathbf{Sc}_{sclqp}^{maprc}]. \quad \{22\}$$

0.2.2.1.1.2. Second step of the process of systematic revealing various aspects of the science: construction of concept-vectors $\& [\mathbf{Sc}^i]_2$ and $\& [\mathbf{Sc}_j]_2$, and of concept-matrix $\& [\mathbf{Sc}_j^i]_{2,2}$ of possible formal explications of the concept "science"

On second step of the process of semantic analysis of the concept "science", taking into account the results of interpretation of the components of matrix {4} $\& [\mathbf{Sc}_j^i]_1$ exposed in the section 0.2.2.1.1.1, we shall construct a pair of more detailed concept-vectors $\& [\mathbf{Sc}^i]_2$ and $\& [\mathbf{Sc}_j]_2$:

$$\& [\mathbf{Sc}^i]_2 = \begin{bmatrix} \mathbf{Sc}^{crp} \\ \mathbf{Sc}^{miprc} \\ \mathbf{Sc}^{maprc} \end{bmatrix}, \quad \{23\}$$

and

$$\& [\mathbf{Sc}_j]_2 = \& [\mathbf{Sc}_{hekn} \mathbf{Sc}_{htknl} \mathbf{Sc}_{hpknl} \mathbf{Sc}_{knlsbj} \mathbf{Sc}_{knlmns} \mathbf{Sc}_{sclknl} \mathbf{Sc}_{sclind} \mathbf{Sc}_{sclgrp} \mathbf{Sc}_{sclqp}], \quad \{24\}$$

Their Cartesian product gives a second, more detailed matrix of the type $\& [\mathbf{Sc}_j^i]_{2,2}$:

$$\& [\mathbf{Sc}_j^i]_{2,2} = \begin{bmatrix} \mathbf{Sc}^{crp} \\ \mathbf{Sc}^{miprc} \\ \mathbf{Sc}^{maprc} \end{bmatrix} \otimes \& [\mathbf{Sc}_{hekn} \mathbf{Sc}_{htknl} \mathbf{Sc}_{hpknl} \mathbf{Sc}_{knlsbj} \mathbf{Sc}_{knlmns} \mathbf{Sc}_{sclknl} \mathbf{Sc}_{sclind} \mathbf{Sc}_{sclgrp} \mathbf{Sc}_{sclqp}] =$$

$$= \begin{matrix} & \begin{bmatrix} \text{Sc}_{heknl}^{crp} & \text{Sc}_{htknl}^{crp} & \text{Sc}_{hpkn}^{crp} & \text{Sc}_{knlsbj}^{crp} & \text{Sc}_{knlmns}^{crp} & \text{Sc}_{sclkn}^{crp} & \text{Sc}_{sclnd}^{crp} & \text{Sc}_{sclgrp}^{crp} & \text{Sc}_{sclgap}^{crp} \\ \text{Sc}_{heknl}^{miprc} & \text{Sc}_{htknl}^{miprc} & \text{Sc}_{hpkn}^{miprc} & \text{Sc}_{knlsbj}^{miprc} & \text{Sc}_{knlmns}^{miprc} & \text{Sc}_{sclkn}^{miprc} & \text{Sc}_{sclnd}^{miprc} & \text{Sc}_{sclgrp}^{miprc} & \text{Sc}_{sclgap}^{miprc} \\ \text{Sc}_{heknl}^{maprc} & \text{Sc}_{htknl}^{maprc} & \text{Sc}_{hpkn}^{maprc} & \text{Sc}_{knlsbj}^{maprc} & \text{Sc}_{knlmns}^{maprc} & \text{Sc}_{sclkn}^{maprc} & \text{Sc}_{sclnd}^{maprc} & \text{Sc}_{sclgrp}^{maprc} & \text{Sc}_{sclgap}^{maprc} \end{bmatrix} \\ \& \end{matrix} \quad \{25\}$$

0.2.2.1.1.3. Third step of the process of systematic revealing various aspects of the science: construction of concept-vector $\& [\text{Sc}^i]_3$ and of concept-matrix $\& [\text{Sc}^i]_{3,2}$ of possible formal explications of the concept “science”

0.2.2.1.1.3.1. The science treated as process of functioning the corpus of scientific knowledge together with of the immediate factors for its production $\text{Sc}_{knl+ifctr}^{crp}$ on macrolevel in accordance with section 0.2.2.1.1.1, item “2.3.2)”, is a process of historical development (in the broad sense including also stages or phases of decay) of “ $\text{Sc}_{knl+ifctr}^{crp}$ ” in a large scale of time. Such a timing of our discourse about science requires introducing some special designations. Signs “[” and “(” will denote bounds of left closed and of left open time intervals, respectively; by analogy with this just mentioned case signs “]” and “)” will denote bounds of right closed and of right open time intervals, respectively. If $x \neq y$, then “(x, y)”, “(x, y]”, “[x, y)”, and “[x, y]” denote continuous interval beginning from x and ending in y. So, the symbolic expressions “ $\text{Sc}^{maprc}(x, y)$ ” and “ $\text{Sc}^{maprc}[x, y]$ ” will denote “historical development of science during (open or closed, respectively) interval between moments of time x and y”, where x and y are variables; also the variants $\text{Sc}^{maprc}[x, y)$ and $\text{Sc}^{maprc}(x, y]$ are allowed. Using constants a, b, etc. which denote some fixed moments of time and are defined by means of fixing a datum or an event in the history of science (for example the time of a given scientific publication), we may write the symbolic expressions “ $\text{Sc}^{maprc}(a, b)$ ”, “ $\text{Sc}^{maprc}[a, b]$ ”, “ $\text{Sc}^{maprc}[a, b)$ ” and “ $\text{Sc}^{maprc}(a, b]$ ” with analogical meaning however concerning some fixed moments of time. Naturally, various variations as one of the two letters to denote constant and the other to denote variable, are also allowed. Besides the general concepts of historical development of science during a time interval is treated in such a way that the following two pairs of additive-axioms and incompatibility-axioms expressed by means of using some well known symbols of the set theory and of the classical propositional logic are accepted as valid:

$$\begin{aligned} \text{Ax. Scad. I. } ([x, y] = [x, z] \cup [z, y]) &\rightarrow \\ &\rightarrow (\text{Sc}^{maprc}[x, y] = \text{Sc}^{maprc}[x, z] \cup \text{Sc}^{maprc}[z, y]) . \end{aligned} \quad \{26\}$$

$$\begin{aligned} \text{Ax. Scad. II. } ([x, y] = [x, z] \cup (z, y)) &\rightarrow \\ &\rightarrow (\text{Sc}^{maprc} [x, y] = \text{Sc}^{maprc} [x, z] \cup \text{Sc}^{maprc} (z, y)) . \end{aligned} \quad \{27\}$$

$$\text{Ax. Scinc. I. } ([x, y] \cap [u, v] = \emptyset) \rightarrow (\text{Sc}^{maprc} [x, y] \cap \text{Sc}^{maprc} [u, v] = \emptyset) \{28\}$$

$$\text{Ax. Scinc. II. } ([x, y] \cap (u, v) = \emptyset) \rightarrow (\text{Sc}^{maprc} [x, y] \cap \text{Sc}^{maprc} (u, v) = \emptyset) \{29\}$$

$$t \in [x, y] \quad \{30\}$$

means “moment of time in the closed time interval between moments of time x and y , including last ones” (we will have analogical cases if in the formula {30} one or both closed bounds will be replaced with open ones).

Let we assume by definition following two equations:

$$\text{Df. } [x] \quad [x] = [x, x] \quad \{31\}$$

$$\text{Df. } \text{Sc}^{maprc} \quad \text{Sc}^{maprc} = \text{Sc}^{maprc} [v, w] \quad \{32\}$$

Let τ means “present moment of time”, and let we assume further explication of the concepts “past historical development of science”, “present state of science” and “future development of science” by means of the symbolic expressions “ $\text{Sc}^{maprc} [v, \tau]$ ”, “ $\text{Sc}^{maprc} [\tau]$ ” and “ $\text{Sc}^{maprc} (\tau, w)$ ”, respectively, where “ v ” denotes initiation of human knowledge, and “ w ” – arbitrary remote moment of time in the future. Now we may formulate and prove the following thesis:

$$\begin{aligned} \text{Th. 1. } (\text{Sc}^{maprc} = \text{Sc}^{maprc} [v, \tau] \cup \text{Sc}^{maprc} [\tau] \cup \text{Sc}^{maprc} (\tau, w)) \& \quad \{33\} \\ \& (\text{Sc}^{maprc} [v, \tau] \cap \text{Sc}^{maprc} [\tau] = \emptyset) \& (\text{Sc}^{maprc} [\tau] \cap \text{Sc}^{maprc} (\tau, w) = \emptyset) \end{aligned}$$

Proof:

Let we assume that “ v ” will denote beginning of human knowledge, and “ w ” – arbitrary remote moment of time in the future.

- | | |
|--|---|
| (1) $[x, z] = [x, y] \cup [y, z]$ | Thesis of set theory |
| (2) $[v, \tau] = [v, \tau] \cup [\tau, \tau]$ | From (1) by substitution
$x \rightarrow v, y \rightarrow \tau, z \rightarrow \tau.$ |
| (3) $([v, \tau] = [v, \tau] \cup [\tau, \tau]) \rightarrow$
$\rightarrow (\text{Sc}^{maprc} [v, \tau] = \text{Sc}^{maprc} [v, \tau] \cup \text{Sc}^{maprc} [\tau, \tau])$ | From Ax. Scad I by substitution
$x \rightarrow v, y \rightarrow \tau, z \rightarrow \tau.$ |
| (4) $\text{Sc}^{maprc} [v, \tau] = \text{Sc}^{maprc} [v, \tau] \cup \text{Sc}^{maprc} [\tau, \tau]$ | From (2) and (3) by modus ponens |

- (5) $[x, z] = [x, y] \cup (y, z)$ Thesis of set theory
- (6) $[v, w] = [v, \tau] \cup (\tau, w)$ From (5) by substitution
 $x \rightarrow v, y \rightarrow \tau, z \rightarrow w.$
- (7) $([v, w] = [v, \tau] \cup (\tau, w)) \rightarrow$ From Ax. Scad II by substitution
 $\rightarrow (Sc^{maprc} [v, w] = Sc^{maprc} [v, \tau] \cup Sc^{maprc} (\tau, w))$ $x \rightarrow v, y \rightarrow w, z \rightarrow \tau.$
- (8) $Sc^{maprc} [v, w] = Sc^{maprc} [v, \tau] \cup Sc^{maprc} (\tau, w)$ From (6) and (7) by modus ponens
- (9) $Sc^{maprc} [v, w] =$ By replacement of $Sc^{maprc} [v, \tau]$ in
 $= Sc^{maprc} [v, \tau] \cup Sc^{maprc} [\tau, \tau] \cup Sc^{maprc} (\tau, w)$ (8) with right part of (4).
- (10) $[\tau] = [\tau, \tau]$ From Df. $[x]$ by substitution
 $x \rightarrow \tau.$
- (11) $Sc^{maprc} [v, w] =$ By replacement in (9) of right part
 $= Sc^{maprc} [v, \tau] \cup Sc^{maprc} [\tau] \cup Sc^{maprc} (\tau, w)$ of (10) with its left part.
- (12) $Sc^{maprc} = Sc^{maprc} [v, \tau] \cup Sc^{maprc} [\tau] \cup Sc^{maprc} (\tau, w)$ From (11) by replacement of
 $Sc^{maprc} [v, w]$ with Sc^{maprc}
according to Df. Sc^{maprc} .
- (13) $[v, \tau] \cap [\tau, \tau] = \emptyset$ According to the definition of open
set $7(\tau \in [v, \tau])$; also $[\tau, \tau]$ being equal
to $[\tau]$, by definition contains only one
element, namely τ , which as it was
established above is not included in
 $[v, \tau)$, consequently no element of $[v, \tau)$
is included in $[\tau]$, respectively in $[\tau, \tau]$.
- (14) $([v, \tau] \cap [\tau, \tau] = \emptyset) \rightarrow$ From Ax.Scinc.I by substitutions
 $\rightarrow (Sc^{maprc} [v, \tau] \cap Sc^{maprc} [\tau, \tau] = \emptyset)$ $x \rightarrow v, y \rightarrow \tau, u \rightarrow \tau, v \rightarrow \tau.$
- (15) $Sc^{maprc} [v, \tau] \cap Sc^{maprc} [\tau, \tau] = \emptyset$ From (13) and (14) by modus ponens.
- (16) $Sc^{maprc} [v, \tau] \cap Sc^{maprc} [\tau] = \emptyset$ By replacement of right part of (10)
with its left part in (15).
- (17) $[\tau, \tau] \cap (\tau, w) = \emptyset$ According to considerations analogical
to ones formulated concerning (13).
- (18) $([\tau, \tau] \cap (\tau, w) = \emptyset) \rightarrow$ From Ax.Scinc.II by substitutions
 $\rightarrow (Sc^{maprc} [\tau, \tau] \cap Sc^{maprc} (\tau, w) = \emptyset)$ $x \rightarrow \tau, y \rightarrow \tau, u \rightarrow \tau, v \rightarrow w.$
- (19) $Sc^{maprc} [\tau, \tau] \cap Sc^{maprc} (\tau, w) = \emptyset$ From (17) and (18) by modus ponens.
- (20) $Sc^{maprc} [\tau] \cap Sc^{maprc} (\tau, w) = \emptyset$ By replacement of right part of (10)

$$(20) p \rightarrow (q \rightarrow (r \rightarrow (p \& q \& r)))$$

$$(21) (Sc^{maprc} [v, w] = Sc^{maprc} [v, \tau] \cup Sc^{maprc} [\tau] \cup Sc^{maprc} (\tau, w)) \rightarrow \\ \rightarrow ((Sc^{maprc} [v, \tau] \cap Sc^{maprc} [\tau] = \emptyset) \rightarrow \\ \rightarrow ((Sc^{maprc} [\tau] \cap Sc^{maprc} (\tau, w) = \emptyset) \rightarrow \\ \rightarrow ((Sc^{maprc} [v, w] = Sc^{maprc} [v, \tau] \cup Sc^{maprc} [\tau] \cup Sc^{maprc} (\tau, w)) \& \\ \& (Sc^{maprc} [v, \tau] \cap Sc^{maprc} [\tau] = \emptyset) \& \\ \& (Sc^{maprc} [\tau] \cap Sc^{maprc} (\tau, w) = \emptyset))))$$

$$(22) (Sc^{maprc} [v, \tau] \cap Sc^{maprc} [\tau] = \emptyset) \rightarrow \\ \rightarrow ((Sc^{maprc} [\tau] \cap Sc^{maprc} (\tau, w) = \emptyset) \rightarrow \\ \rightarrow ((Sc^{maprc} [a, w] = Sc^{maprc} [a, \tau] \cup Sc^{maprc} [\tau] \cup Sc^{maprc} (\tau, w)) \& \\ \& (Sc^{maprc} [v, \tau] \cap Sc^{maprc} [\tau] = \emptyset) \& \\ \& (Sc^{maprc} [\tau] \cap Sc^{maprc} (\tau, w) = \emptyset))))$$

$$(23) (Sc^{maprc} [\tau] \cap Sc^{maprc} (\tau, w) = \emptyset) \rightarrow \\ \rightarrow ((Sc^{maprc} [v, w] = Sc^{maprc} [v, \tau] \cup Sc^{maprc} [\tau] \cup Sc^{maprc} (\tau, w)) \& \\ \& (Sc^{maprc} [v, \tau] \cap Sc^{maprc} [\tau] = \emptyset) \& \\ \& (Sc^{maprc} [\tau] \cap Sc^{maprc} (\tau, w) = \emptyset))$$

$$(24) (Sc^{maprc} [v, w] = Sc^{maprc} [v, \tau] \cup Sc^{maprc} [\tau] \cup Sc^{maprc} (\tau, w)) \& \\ \& (Sc^{maprc} [v, \tau] \cap Sc^{maprc} [\tau] = \emptyset) \& \\ \& (Sc^{maprc} [\tau] \cap Sc^{maprc} (\tau, w) = \emptyset)$$

with its left part in (19).

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From (20) by substitutions

$$p \rightarrow Sc^{maprc} [a, w] = Sc^{maprc} [a, \tau] \cup Sc^{maprc} [\tau] \cup Sc^{maprc} (\tau, w), \\ q \rightarrow Sc^{maprc} [a, \tau] \cap Sc^{maprc} [\tau] = \emptyset, \\ r \rightarrow Sc^{maprc} [\tau] \cap Sc^{maprc} (\tau, w) = \emptyset.$$

From (11) and (21) by modus ponens.

.

From (16) and (22) by modus ponens.

.

From (20) and (23) by modus ponens.

The last formula (24) coincides with Th.1; consequently Th.1 is proved by means of the succession of formulae (1)-(24) above.

Note to Th.1. One may say that roughly speaking Th.1 reads as follows: in the historical process of development of science a sharp distinction between three mutually incompatible parts takes place: **(1)** past historical development of science, **(2)** present state of science, and **(3)** future process of historical development of science.

It may turn out to be useful to pay attention to the circumstance that these words are open to misconstruction: in view of the fact that such a distinction is obvious from an intuitive point of view one can say that using the means of symbolic logic in philosophy of science is unproductive. In this connection I have to formulate following three items:

1) First of all it is necessary to be stressed that Thesis 1 is not reducible to its rough verbal expression - it is a definite explication of it, i.e. a definite choice from the set of the various possible explications.

2) On the second place we have to take into account that Thesis 1 taken together with its proof (which is possible only in consequence of the explication mentioned above, in item “1)”) reveals its logical presuppositions, otherwise speaking it reveals the conditions which are necessary such a distinction to take place.

3) On the third place we have to take into consideration the fact that at the very outset of exposition of a mathematical theory, for example of the differential and integral calculus, we have to prove in a strict way trivial, obvious truths, however there accumulation prepare the ground to discover and prove in a strict way further nontrivial propositions. It is not excluded the analogical case to take place also in a well developed axiomatic theory of the history of science from which only some very initial elements were formulated above; moreover it is quite possible to construct various axiomatic theories of the history of science like construction of various geometries after elaboration of Euclidian geometry.

0.2.2.1.1.3.2. The time-intervals attached to the concepts of science in the symbolic expressions $Sc[x, y]$, $Sc[x, y)$ and so on are definite quantitatively – by means of definiteness of the first and last moment of time. The theoretical treatment of the historical process of development of science (which particularly is significant for the purposes of philosophic-methodological investigations) requires introducing into language of science more specified concepts for time intervals which have to be distinguished by means of qualitative definiteness. And so, we shall introduce here the following definition:

Df. u. **(1)** A time-interval of historical development of science which is distinguished as a whole by means of given qualitative definiteness will be called *historical unite (of time)* and will be denoted as “**u**”; besides **(2)** the symbol “**Sc u**” will mean by definition “science (respectively, historical development of science) during the historical unit of time **u**”; **(3)** the symbolic expression $u[x, y]$ will mean by definition “historical unite of time beginning at the moment of time **x** and ending at the moment of time **y**” (we will have analogical case if in the

given symbolic expression one or both closed bounds will be replaced with open ones), and **(4)** $u[\tau]$ will mean by definition “the historical unite of time containing the present moment of time”, or otherwise speaking, “present historical unite of time”

Taking into account the results of research in the field of history of science, partially revealed in section 0.2.1, we shall accept the historical unites of time of the historical development of science to be periods, stages, phases, and eventually next smaller historical time units, i.e. we accept to be valid the following element-inclusion:

$$u \in \{\text{period, stage, phase, eventually next smaller historical time units}\}, \quad \{34\}$$

besides the set of elements which is put in the $\{, \}$ -brackets is treated as ordered one in the sense that if the various kinds of historical time unites of the development of science are treated as corresponding continuous sets of moments of time, than we have the following “strict set-inclusion”:

$$\text{phase} \subset \text{stage} \subset \text{period}, \quad \{35\}$$

besides by definition of the concept “strict set-inclusion”:

$$\text{Df. } \subset. \quad (X \subset Y) \rightarrow \neg(Y \subset X) \quad \{36\}$$

(If there are also historical units of time smaller than phase, then analogical strict set-inclusion takes place for them too.)

Note to section 0.2.2.1.1.3.2. For example in section 0.2.1 the fact 2 demonstrates the period of pre-scientific knowledge, preparing the ground for origination of sciences; the facts 4-12 in their totality and connections outline the period of ancient scientific thought, besides facts 3-7, 9, and 12 in their totality and connections outline the period of ancient mathematics, consisting of four stages: **(1)** the stage of initial formation of mathematics as independent field of knowledge – from Thales to Pythagoreans, facts 3-7, **(2)** the stage of formation of axiomatic method and theories in Plato’s Academy, fact 9, **(3)** the stage of elaboration of mathematics as unified axiomatic system by Euclid, fact 12, items 1-3, and **(4)** the stage of further development of mathematics after its unified, systematic axiomatic construction during the ancient epoch, fact 12, item 4. In the framework of the first stage of the first (ancient) period of the history of mathematics one may distinguish between three phases **(1*)** the phase of Miletusean school of mathematics – Thales, Anaximander, etc., facts 4 and 5, **(2*)** the phase of Pythagorean school of mathematics during the life of Pythagoras, fact 6, **(3*)** the phase of Pythagorean school of mathematics after the death of Pythagoras, fact 7. The various items of fact 14 also give a hint for analogical structuralization of time unites of various degrees of development of sciences during the period beginning with the epoch of Renaissance

and ending with publication of Norbert Wiener's "Cybernetics", which however we shall avoid here to realize explicitly in order to shorten the exposition (in accordance with the requirement for abridged and simplified presentation of the present topic formulated in section 0.1.2).

0.2.2.1.1.3.3. Now we can formulate explications of the three more concrete kinds of concept "process of historical development of science" - **(1)** "present state of science", **(2)** "history of science", and **(3)** "future development of science":

1) The concept "present state of science" **(1)** used here about science in both senses: **(1.1)** about science as a whole - "present state of science as a whole" as well as **(1.2)** about a given part or a given aspect of science - "present state of mathematics", "present state of algebra", "present state of group theory", "present state of axiomatic method of mathematics", "present state of physics", "present state of thermodynamics", "present state of application of logic to theoretical physics", etc., and **(2)** related to a given type of historical unit of time of development of science as **(2.1)** to the present period of development of science, **(2.2)** to the present stage of development of science, **(2.3)** to the present phase of development of science, and so on, **(3)** will be denoted by " $\mathbf{Sc}^{\text{prsstur}}$ ", where " $\mathbf{u}$ " means the type of historical unit of time to which the term "present state" is related and " τ " means the present moment of time. The content of this concept consists of a set of characteristic features of the science, including characteristic elements of its content, which of their totality set its qualitative definiteness during the given historical unite (i.e. period, stage, phase, etc.). So, the following equality is valid by definition:

Df. $\mathbf{Sc}^{\text{prsstur}}$ $\mathbf{Sc}^{\text{prsstur}} = (\mathbf{Sc}^{\text{maprc}}_{\mathbf{u}}[\mathbf{a}, \mathbf{b}] \ \& \ (\tau \in_{\mathbf{u}}[\mathbf{a}, \mathbf{b}]))$, provided $\mathbf{u} \in \{\text{stage, phase}\}$.
{37}

Note to Df. $\mathbf{Sc}^{\text{prsstur}}$. Do not mix this explication of the concept "present state of science" denoted by $\mathbf{Sc}^{\text{prsstur}}$ and presupposing structuralization of the macroprocess of development of science by means of historical units of time like periods, stages, phases with the explication denoted above by " $\mathbf{Sc}^{\text{maprc}}[\tau]$ ".

For the purposes of epistemological and methodological investigations it is significant to distinguish between following two parts of a given present state of science which may be treated as correspondingly two more detailed aspects of the concept "science":

1.1) The science during time interval $[\mathbf{a}, \tau]$, provided $(\tau \in_{\mathbf{u}}[\mathbf{a}, \mathbf{b}])$, and $\mathbf{u} \in \{\text{stage, phase}\}$, will be named "effective part of the present state of science" and denoted by " $\mathbf{Sc}^{\text{efprsstur}}$ ". So, by definition we have:

Df. $Sc^{efprsstur}$ $Sc^{efprsstur} = (\tau \in_u [a, b] \ \& \ Sc^{maprc} [a, \tau])$, provided $u \in \{\text{stage, phase}\}$ {38}

1.2) The science during the time interval $(\tau, b]$, provided the conditions mentioned above, will be named “non-effective part of the present state of science” and will be denoted by “ $Sc^{nefprsstur}$ ”. So, by definition we have:

Df. $Sc^{nefprsstur}$ $Sc^{nefprsstur} = (\tau \in_u [a, b] \ \& \ Sc^{maprc} (\tau, b))$, provided $u \in \{\text{stage, phase}\}$ {39}

2) The concept “history of science” (cf. the explanation about using the term “science” on the similar place in item 1 of the present section), related to a given type of historical units of time of the process of its development (as, for example, to periods, to stages, to phases, etc., will be denoted as “ Sc^{hst} ” and will mean “the process of development of science from its beginning (involving also the period of prehistory of science) till the beginning of a given contemporary historical unit of time”. So, we have by definition:

Df. Sc^{hstur} $Sc^{hstur} = (\tau \in_u [a, b] \ \& \ Sc [v, a])$ {40}

3) The concept “future development of science” (cf. the explanation about using the term “science” on the similar place in item “1)”) for a given present moment of time τ - “ $Sc^{ftrdvlt}$ ” means “the process of development of science during the time interval (τ, w) where the future moment of time w in the general case, is a variable quantity.

Df. $Sc^{ftrdvlt}$ $Sc^{ftrdvlt} = Sc^{maprc} (\tau, w)$ {41}

Note to Df. Sc^{hstur} and Df. $Sc^{ftrdvlt}$. In contrast to above explications of concepts “past historical development of science” and “future development of science” denoted by means of the symbolic expressions “ $Sc^{maprc} [v, \tau]$ ” and “ $Sc^{maprc} (\tau, w)$ ”, respectively, their explications here by means of **Df. Sc^{hstur}** and **Df. $Sc^{ftrdvlt}$** (cf. formulae {37} and {41}) presuppose structuralization of the macroprocess of development of science by means of historical units of time like periods, stages, phases (cf. also note to Df. $Sc^{prsstur}$).

From extensional point of view by definition we have in general the following set-theoretical equivalence:

Th. 3. $(Sc^{maprc} = Sc^{hstur} \cup Sc^{efprsstur} \cup Sc^{ftrdvlt}) \ \& \ (Sc^{hstur} \cap Sc^{efprsstur} = \emptyset) \ \& \ (Sc^{efprsstur} \cap Sc^{ftrdvlt} = \emptyset)$ {42}

Note to Th. 3. The proof of Th 3 is like the one of Th. 1 above.

0.2.2.1.1.3.4. In accordance with that what is formulated in the previous section 0.2.2.1.1.3.3 we can construct the following 3-column-conjunctive-concept-vector $\text{[Sc}^{maprc}\text{]}_1$:

$$\&[\mathbf{Sc}^{maprc}]_1 = \begin{bmatrix} \mathbf{Sc}^{\text{hstu}\tau} \\ \mathbf{Sc}^{\text{efprsstu}\tau} \\ \mathbf{Sc}^{\text{ftrdvl}\tau} \end{bmatrix} \quad \{43\}$$

0.2.2.1.1.3.5. Formally replacing the third component “ $\mathbf{Sc}^{\text{maprc}}$ ” of the vector in the right side of equation {23} by means of the right side of equation {43} (or, with regard to the contents: taking into account the revealing of the three mutually complementary more concrete kinds of the concept “process of historical development of science” in section 0.2.2.1.1.3.3) we can split it into three new separate components of the vector just mentioned and in such a way we can obtain a more detailed 5-column-conjunctive-concept-vector $\&[\mathbf{Sc}^i]_3$:

$$\&[\mathbf{Sc}^i]_3 = \begin{bmatrix} \mathbf{Sc}^{\text{crp}} \\ \mathbf{Sc}^{\text{miprc}} \\ \mathbf{Sc}^{\text{hstu}\tau} \\ \mathbf{Sc}^{\text{efprsstu}\tau} \\ \mathbf{Sc}^{\text{ftrdvl}\tau} \end{bmatrix} \quad \{44\}$$

0.2.2.1.1.3.6. The Cartesian product of concept-vectors ${}_&[\mathbf{Sc}^i]_3$ (cf. {44}) and ${}_&[\mathbf{Sc}_j]_2$ (cf. {24}) gives us the following below 5×9 – conjunctive-concept-matrix ${}_&[\mathbf{Sc}_j^i]_{3,2}$:

$$\& [\mathbf{Sc}^i_j]_{3,2} = \& [\mathbf{Sc}^i]_3 \otimes \& [\mathbf{Sc}_j]_2 = \begin{bmatrix} \mathbf{Sc}^{\text{crp}} \\ \mathbf{Sc}^{\text{miprc}} \\ \mathbf{Sc}^{\text{hstu}\tau} \\ \mathbf{Sc}^{\text{efprstu}\tau} \\ \mathbf{Sc}^{\text{ftrdvl}\tau} \end{bmatrix} \otimes$$

$$\otimes_{\&} [\text{Sc}_{hekl} \text{ Sc}_{htkl} \text{ Sc}_{hpkl} \text{ Sc}_{knlslj} \text{ Sc}_{knlms} \text{ Sc}_{sclkl} \text{ Sc}_{sclnd} \text{ Sc}_{sclgrp} \text{ Sc}_{sclgrp}] =$$

$$= \begin{matrix} & \begin{matrix} \text{Sc}_{\text{heknl}}^{\text{crp}} & \text{Sc}_{\text{htknl}}^{\text{crp}} & \text{Sc}_{\text{hpknl}}^{\text{crp}} & \text{Sc}_{\text{knlsbj}}^{\text{crp}} & \text{Sc}_{\text{knlmns}}^{\text{crp}} & \text{Sc}_{\text{sclknl}}^{\text{crp}} & \text{Sc}_{\text{sclind}}^{\text{crp}} & \text{Sc}_{\text{sclgrp}}^{\text{crp}} & \text{Sc}_{\text{sclcap}}^{\text{crp}} \end{matrix} \\ \begin{matrix} \text{Sc}_{\text{heknl}}^{\text{miprc}} & \text{Sc}_{\text{htknl}}^{\text{miprc}} & \text{Sc}_{\text{hpknl}}^{\text{miprc}} & \text{Sc}_{\text{knlsbj}}^{\text{miprc}} & \text{Sc}_{\text{knlmns}}^{\text{miprc}} & \text{Sc}_{\text{sclknl}}^{\text{miprc}} & \text{Sc}_{\text{sclind}}^{\text{miprc}} & \text{Sc}_{\text{sclgrp}}^{\text{miprc}} & \text{Sc}_{\text{sclcap}}^{\text{miprc}} \\ \text{Sc}_{\text{heknl}}^{\text{hstut}} & \text{Sc}_{\text{htknl}}^{\text{hstut}} & \text{Sc}_{\text{hpknl}}^{\text{hstut}} & \text{Sc}_{\text{knlsbj}}^{\text{hstut}} & \text{Sc}_{\text{knlmns}}^{\text{hstut}} & \text{Sc}_{\text{sclknl}}^{\text{hstut}} & \text{Sc}_{\text{sclind}}^{\text{hstut}} & \text{Sc}_{\text{sclgrp}}^{\text{hstut}} & \text{Sc}_{\text{sclcap}}^{\text{hstut}} \\ \text{Sc}_{\text{heknl}}^{\text{efprsstur}} & \text{Sc}_{\text{htknl}}^{\text{efprsstur}} & \text{Sc}_{\text{hpknl}}^{\text{efprsstur}} & \text{Sc}_{\text{knlsbj}}^{\text{efprsstur}} & \text{Sc}_{\text{knlmns}}^{\text{efprsstur}} & \text{Sc}_{\text{sclknl}}^{\text{efprsstur}} & \text{Sc}_{\text{sclind}}^{\text{efprsstur}} & \text{Sc}_{\text{sclgrp}}^{\text{efprsstur}} & \text{Sc}_{\text{sclcap}}^{\text{efprsstur}} \\ \text{Sc}_{\text{heknl}}^{\text{ftrdvlt}} & \text{Sc}_{\text{htknl}}^{\text{ftrdvlt}} & \text{Sc}_{\text{hpknl}}^{\text{ftrdvlt}} & \text{Sc}_{\text{knlsbj}}^{\text{ftrdvlt}} & \text{Sc}_{\text{knlmns}}^{\text{ftrdvlt}} & \text{Sc}_{\text{sclknl}}^{\text{ftrdvlt}} & \text{Sc}_{\text{sclind}}^{\text{ftrdvlt}} & \text{Sc}_{\text{sclgrp}}^{\text{ftrdvlt}} & \text{Sc}_{\text{sclcap}}^{\text{ftrdvlt}} \end{matrix} \end{matrix} \quad \{45\}$$

We can obtain the semantic interpretation of the components Sc_j^i of the 5×9 -conjunctive-concept-matrix on the right site of equation {45} by means of combination in an appropriate way of the already given above semantic interpretation of corresponding components Sc^i and Sc_j of the concept-vectors $\& [\text{Sc}^i]_3$ and $\& [\text{Sc}_j]_2$, respectively.

0.2.2.1.1.4. Fourth step of the process of systematic revealing various aspects of the science: construction of concept-vector $\& [\text{Sc}^i]_4$ and of concept-matrix $\& [\text{Sc}_j^i]_{4,2}$ of possible formal explications of the concept “science”

0.2.2.1.1.4.1. Let us be given $\tau \in \llbracket \mathbf{a}, \mathbf{b} \rrbracket$, $\mathbf{u} \in \{\text{stage, phase}\}$. The time-interval $[\tau, \mathbf{w})$, where “ $\mathbf{w}$ ” denotes arbitrary distant moment of time, besides in any case it will be presupposed that time-moment $\mathbf{w}$ follows after time-moment $\mathbf{b}$, by means of the last one can be split into two parts in particular significant for the epistemological and methodological investigations, inducing correspondingly two more concrete kinds of the concept “future development of science” denoted by $\text{Sc}^{\text{ftrdvlt}}$.

1) The process of development of science during immediate future, denoted by “ $\text{Sc}^{\text{imftrdvlt}}$ ”, consists in the future development of science in the framework of the given present state (present stage or phase) of science embracing the period of time immediately following after the present moment of time. So, from extensional point of view we have by definition:

$$\text{Df. } \text{Sc}^{\text{imftrdvlt}} \cdot \text{Sc}^{\text{imftrdvlt}} = (\tau \in \llbracket \mathbf{a}, \mathbf{b} \rrbracket \ \& \ \text{Sc}(\tau, \mathbf{b})), \text{ provided } \mathbf{u} \in \{\text{stage, phase}\} \quad \{46\}$$

$$\text{Th. 2. } \quad \text{Sc}^{\text{imftrdvlt}} = \text{Sc}^{\text{efprsstur}} \quad \{47\}$$

Proof. Thesis 2 follows immediately from formulae {39} and {46} by the rule of inference:

$$\text{If: } \quad \mathbf{A} = \mathbf{C}$$

$$\text{then } \frac{\mathbf{B} = \mathbf{C}}{\mathbf{A} = \mathbf{B}}$$

Note to item “1)” of section 0.2.2.1.1.4.1. Thesis 2 formulated by means of formula {47} expresses only the extensional aspect of the relation of equality of both explications of concept “science”, denoted by “ $\mathbf{Sc}^{\text{nefprsstur}}$ ” and “ $\mathbf{Sc}^{\text{imftrdvlur}}$ ”. It is worthy to be stressed that being extensionally identical as embracing one and the same interval of time the both explications are intentionally different by virtue of their different direction – the explication denoted by “ $\mathbf{Sc}^{\text{nefprsstur}}$ ” is directed to the present moment of time τ , while the explication denoted by “ $\mathbf{Sc}^{\text{imftrdvlur}}$ ” is directed to the future moment of time.

2) Science during distant time in the future (in the remote future) of its development, denoted by “ $\mathbf{Sc}^{\text{dsftrdvlur}}$ ”, consists in the future development (evolution) of science following after its present state (respectively after its present stage or its present phase) of development. So, we have by definition:

Df. $\mathbf{Sc}^{\text{dsftrdvlur}}$. $\mathbf{Sc}^{\text{dsftrdvlur}} = (\tau \in_u [\mathbf{a}, \mathbf{b}] \ \& \ \mathbf{Sc}(\mathbf{b}, \mathbf{w}))$, provided $\mathbf{u} \in \{\text{stage, phase}\}$ {48}

0.2.2.1.1.4.2. The splitting of concept of science treated as a future process of development and denoted by “ $\mathbf{Sc}^{\text{ftrdvlur}}$ ” mentioned in section 0.2.2.1.1.4.1 can be expressed symbolically by means of the following 2-column-conjunctive-concept-vector:

$$\&[\mathbf{Sc}^{\text{ftrdvlur}}]_1 = \&\begin{bmatrix} \mathbf{Sc}^{\text{imftrdvlur}} \\ \mathbf{Sc}^{\text{dsftrdvlur}} \end{bmatrix} \quad \{49\}$$

0.2.2.1.1.4.3. Formally replacing the fifth component “ $\mathbf{Sc}^{\text{ftrdvlur}}$ ” of the vector in the right side of equation {44} by means of the right side of equation {49} (or, with regard to the contents: taking into account the revealing of two mutually complementary more concrete kinds of the concept “process of future development of science”, or, otherwise speaking “science as a future process of development (in the broad sense including also decay)”) in section 0.2.2.1.1.4.1 we can split it into two new separate components of the vector just mentioned and in such a way we can obtain a more detailed 6-column-conjunctive-concept-vector $\&[\mathbf{Sc}^i]_4$:

$${}_&[\mathbf{Sc}^i]_4 = \begin{bmatrix} \mathbf{Sc}^{\text{crp}} \\ \mathbf{Sc}^{\text{miprc}} \\ \mathbf{Sc}^{\text{hstut}} \\ \mathbf{Sc}^{\text{efprsstur}} \\ \mathbf{Sc}^{\text{imftrdvlut}} \\ \mathbf{Sc}^{\text{dsftrdvlut}} \end{bmatrix} \quad \{50\}$$

0.2.2.1.1.4.4. Using the conjunctive-concept-vector ${}_&[\mathbf{Sc}^i]_4$ represented in the formula {50} we can obtain a more detailed conjunctive-concept-matrix ${}_&[\mathbf{Sc}^i_j]$.

0.2.2.1.1.4.4.1. The Cartesian product of concept-vectors ${}_&[\mathbf{Sc}^i]_4$ (cf. {50}) and ${}_&[\mathbf{Sc}_j]_2$ cf. {24}) gives us the following below 6×9 – conjunctive-concept-matrix ${}_&[\mathbf{Sc}^i_j]_{4,2}$:

$${}_&[\mathbf{Sc}^i_j]_{4,2} = {}_&[\mathbf{Sc}^i]_4 \otimes {}_&[\mathbf{Sc}_j]_2 = \begin{bmatrix} \mathbf{Sc}^{\text{crp}} \\ \mathbf{Sc}^{\text{miprc}} \\ \mathbf{Sc}^{\text{hstut}} \\ \mathbf{Sc}^{\text{efprsstur}} \\ \mathbf{Sc}^{\text{imftrdvlut}} \\ \mathbf{Sc}^{\text{dsftrdvlut}} \end{bmatrix} \otimes \begin{bmatrix} \mathbf{Sc}_{\text{hekn}} & \mathbf{Sc}_{\text{htkn}} & \mathbf{Sc}_{\text{hpkn}} & \mathbf{Sc}_{\text{knlsbj}} & \mathbf{Sc}_{\text{knlmns}} & \mathbf{Sc}_{\text{sclkn}} & \mathbf{Sc}_{\text{sclind}} & \mathbf{Sc}_{\text{sclgrp}} & \mathbf{Sc}_{\text{sclgrp}} \end{bmatrix} =$$

$$= \begin{bmatrix} \mathbf{Sc}_{\text{hekn}}^{\text{crp}} & \mathbf{Sc}_{\text{htkn}}^{\text{crp}} & \mathbf{Sc}_{\text{hpkn}}^{\text{crp}} & \mathbf{Sc}_{\text{knlsbj}}^{\text{crp}} & \mathbf{Sc}_{\text{knlmns}}^{\text{crp}} & \mathbf{Sc}_{\text{sclkn}}^{\text{crp}} & \mathbf{Sc}_{\text{sclind}}^{\text{crp}} & \mathbf{Sc}_{\text{sclgrp}}^{\text{crp}} & \mathbf{Sc}_{\text{sclgrp}}^{\text{crp}} \\ \mathbf{Sc}_{\text{hekn}}^{\text{miprc}} & \mathbf{Sc}_{\text{htkn}}^{\text{miprc}} & \mathbf{Sc}_{\text{hpkn}}^{\text{miprc}} & \mathbf{Sc}_{\text{knlsbj}}^{\text{miprc}} & \mathbf{Sc}_{\text{knlmns}}^{\text{miprc}} & \mathbf{Sc}_{\text{sclkn}}^{\text{miprc}} & \mathbf{Sc}_{\text{sclind}}^{\text{miprc}} & \mathbf{Sc}_{\text{sclgrp}}^{\text{miprc}} & \mathbf{Sc}_{\text{sclgrp}}^{\text{miprc}} \\ \mathbf{Sc}_{\text{hekn}}^{\text{hstut}} & \mathbf{Sc}_{\text{htkn}}^{\text{hstut}} & \mathbf{Sc}_{\text{hpkn}}^{\text{hstut}} & \mathbf{Sc}_{\text{knlsbj}}^{\text{hstut}} & \mathbf{Sc}_{\text{knlmns}}^{\text{hstut}} & \mathbf{Sc}_{\text{sclkn}}^{\text{hstut}} & \mathbf{Sc}_{\text{sclind}}^{\text{hstut}} & \mathbf{Sc}_{\text{sclgrp}}^{\text{hstut}} & \mathbf{Sc}_{\text{sclgrp}}^{\text{hstut}} \\ \mathbf{Sc}_{\text{hekn}}^{\text{efprsstur}} & \mathbf{Sc}_{\text{htkn}}^{\text{efprsstur}} & \mathbf{Sc}_{\text{hpkn}}^{\text{efprsstur}} & \mathbf{Sc}_{\text{knlsbj}}^{\text{efprsstur}} & \mathbf{Sc}_{\text{knlmns}}^{\text{efprsstur}} & \mathbf{Sc}_{\text{sclkn}}^{\text{efprsstur}} & \mathbf{Sc}_{\text{sclind}}^{\text{efprsstur}} & \mathbf{Sc}_{\text{sclgrp}}^{\text{efprsstur}} & \mathbf{Sc}_{\text{sclgrp}}^{\text{efprsstur}} \\ \mathbf{Sc}_{\text{hekn}}^{\text{imftrdvlut}} & \mathbf{Sc}_{\text{htkn}}^{\text{imftrdvlut}} & \mathbf{Sc}_{\text{hpkn}}^{\text{imftrdvlut}} & \mathbf{Sc}_{\text{knlsbj}}^{\text{imftrdvlut}} & \mathbf{Sc}_{\text{knlmns}}^{\text{imftrdvlut}} & \mathbf{Sc}_{\text{sclkn}}^{\text{imftrdvlut}} & \mathbf{Sc}_{\text{sclind}}^{\text{imftrdvlut}} & \mathbf{Sc}_{\text{sclgrp}}^{\text{imftrdvlut}} & \mathbf{Sc}_{\text{sclgrp}}^{\text{imftrdvlut}} \\ \mathbf{Sc}_{\text{hekn}}^{\text{dsftrdvlut}} & \mathbf{Sc}_{\text{htkn}}^{\text{dsftrdvlut}} & \mathbf{Sc}_{\text{hpkn}}^{\text{dsftrdvlut}} & \mathbf{Sc}_{\text{knlsbj}}^{\text{dsftrdvlut}} & \mathbf{Sc}_{\text{knlmns}}^{\text{dsftrdvlut}} & \mathbf{Sc}_{\text{sclkn}}^{\text{dsftrdvlut}} & \mathbf{Sc}_{\text{sclind}}^{\text{dsftrdvlut}} & \mathbf{Sc}_{\text{sclgrp}}^{\text{dsftrdvlut}} & \mathbf{Sc}_{\text{sclgrp}}^{\text{dsftrdvlut}} \end{bmatrix} \quad \{51\}$$

0.2.2.1.1.4.4.2. We can obtain the semantic interpretation of the components $\mathbf{Sc}^i_j$ of the 6×9 -conjunctive-concept-matrix on the right site of equation {51}

by means of combination in an appropriate way of the already given above semantic interpretations of corresponding components $\mathbf{Sc}^i$ and $\mathbf{Sc}_j$ of the concept-vectors $_{\&}[\mathbf{Sc}^i]_4$ and $_{\&}[\mathbf{Sc}_j]_2$, respectively.

0.2.2.1.1.ω. Conclusive remarks concerning conjunctive sets of formal explications of the concept “science” revealing in a systematic way various aspects of the science

0.2.2.1.1.ω.1. Methodological thesis 1. The conjunctive-concept-matrix obtained in the formula {51} is sufficiently appropriate to pass from the formal to contentful (full of mater) explication of the intuitive concept of science on the basis of theoretical analysis of the empirical facts of history of science exposed in section 0.2.1.

0.2.2.1.1.ω.2. Methodological thesis 2. The conjunctive-concept-vectors and matrixes represented in sections 0.2.2.1.1.1-0.2.2.1.1.4 are not the only possible ones - it is possible to produce other ones with like dimension as well as more detailed conjunctive-concept-vectors and matrixes of aspects of the science and in the same time of possible formal explications of the intuitive notion of science.

Note to section 0.2.2.1.1.ω.2. We shall not realise here these possibilities merely because it is not necessary for present methodological investigation – in this case we act up to the requirement for abridgement and simplification of the presentation of analysis of present topic which is contained in the general formulation of the specification of task of present prolegomena formulated in section 0.1.2.

0.2.2.1.1.ω.3. Methodological thesis 3. Every one of the components of the 6×9 -conjunctive-concept-matrix obtained in the equation {51} as well as some components of the conjunctive-concept-matrixes obtained in the equations {25} and {45} as well as some components of the conjunctive-concept-vectors obtained above in the present section 0.2.2.1.1 either are actually subjects or are merely able to be subject of a separate science, respectively of a separate scientific theory concerning the science taken in some its aspect or in some of its aspects. So, one of the components in the 6×9 -conjunctive-concept-matrix obtained in the equation {51} can turn out to be subject of various theories concerning science as well as several components of the same matrix can be together, in their interconnections and totality, subject of a specific theory of science. Besides the concept of science is understood and applied in the given theory of such a kind mainly (but not only) in the sense of corresponding its formal explication or explications.

Examples.

1) At the using the term “science” by the workers in the museums of science, who are an important part of the people working in the field of history of science, predominantly they have in mind namely a set of means of scientific work as measuring instruments, observational and experimental devices, electronic devices for information and word processing, means for record of the results of scientific research and the like in the past stages of the historical development of science, otherwise speaking they have in mind mainly the formal explication of the intuitive notion of science $\text{Sc}_{\text{knlmns}}^{\text{hst}}$.

2) Science in the sense of formal explications $\text{Sc}_{\text{sclind}}^{\text{efprsstt}}$ and $\text{Sc}_{\text{sc lgrp}}^{\text{efprsstt}}$ taken together is subject of sociology of science.

3) Science in the sense of formal explication $\text{Sc}_{\text{hrdknl}}^{\text{miprc}}$ (cf. {14}) is subject of the psychology of scientific research, in particular of scientific creativity.

0.2.2.1.2. Explication of the topic of investigation directed to producing explication of the intuitive concept of science appropriate for the purposes of elaboration of interdisciplinary approach to foundations of science

0.2.2.1.2.1. A topic of scientific investigations is characterised by the definiteness of following two elements:

1) its scope – the set of entities which are enveloped in the given investigation, and

2) its subject – the aspect of the scope mentioned above which is specially subjected to the given investigation and which determines the type of methods of the investigation.

0.2.2.1.2.2. In view of some considerations of general character the following postulation concerning the topic of investigation directed to construction of contentful (full of matter) explication of the intuitive concept of science will be accepted here:

1) The scope of this investigation is the science as a whole; so, all 54 aspects of it revealed systematically in the conjunctive-concept-matrix obtained in the formula {51} are enveloped in some way and in some degree in this investigation.

2) The special subject of this investigation determining in principle the methods of its implementation is the aspect of science to which corresponds the formal explication of its intuitive concept, denoted in the conjunctive-concept-matrix obtained in the formula {51} by “ $\text{Sc}_{\text{htknl}}^{\text{crp}}$ ”. Consequently the rest formal

explications in the just mentioned conjunctive-concept-matrix can be treated and even have to be treated here in such a way and degree in which it is turned out to be indispensable in the course of the process of construction of contentful (full of matter) explication of the concept Sc_{htknl}^{crp} and namely in their connection with such a construction.

0.2.2.1.3. Contentful (full of mater) explication of the concept Sc_{htknl}^{crp}

0.2.2.1.3.1. Empirical treatment of the problem concerning contentful (full of mater) explication of the concept Sc_{htknl}^{crp}

0.2.2.1.3.1.1. On basis of the set of basic facts of the historical process of formation of the characteristic features of sciences represented in section 0.2.1 the following general scheme of the successive steps of natural process of formation of sciences (immediately concerning the process of formation of mathematics however with eventual variations concerning also the process of formation of other sciences) can be constructed:

1) First step: formation and development **(1)** of initial elements of knowledge later entering somehow in the content of the given science, however during this preliminary and preparative stage of its formation they are in the framework either **(1.1)** of the immediate practical knowledge, or **(1.2)** of the philosophy, or **(1.3)** of other branches of the scientific knowledge; otherwise speaking, of initial formation of the scope of research of the given science however in non-proper form and field of knowledge, or **(2)** of essential presuppositions for formation the initial elements of knowledge mentioned above also in the framework either **(2.1)** of the immediate practical knowledge, or **(2.2)** of the philosophy, or **(2.3)** of other branches of the scientific knowledge.

Examples in section 0.2.1: **(1*)** for mathematics – facts 1 and 2, **(2*)** for mechanics – fact 14, item “4.1.2.1.1.2)”, (1) and (2); **(3*)** for chemistry – fact 14, items “4.1.2.2.1)” and “4.1.2.2.2.1)”.

2) Second step: this is a process of initial formation of characteristic **(1)** concepts (correspondingly, objects which are subjected to research), **(2)** problems, **(3)** methods of investigation, **(4)** specific statements, **(5)** total conceptions (possibly debatable and rejected subsequently) **(1)→(5)** of the given science, being in the same time an initial process toward formation of the specific subject of the given science.

Examples in section 0.2.1: **(1*)** for mathematics – facts 3, 4, item “(2)”, 5; **(2*)** for mechanics – fact 14, item “4.1.2.1.1.2)” (3); **(3)** for chemistry – fact 14, item “4.1.2.2.2.2)”.

3) Third step: It is characterised by considerable **(1)** enrichment of the content of given science - with new **(1.1)** concepts (correspondingly, objects which are subjected to research), **(1.2)** problems, **(1.3)** specific statements, **(1.4)** conceptions, **(2)** improvement of its form – **(2.1)** invention of new (especially: fruitful) methods of investigation, **(2.2)** revealing essential connections **(2.2.1)** between different types of concepts (correspondingly, of objects subjected to research) as well as **(2.2.2)** between different specific statements, and **(3)** attention to philosophical meaning of the concrete scientific ideas and methods of research resulting into appearance also of ideas and methods of scientific research which can turn out to be debatable problems however with significant importance for further process of formation of the given field of the science.

Examples in section 0.2.1: **(1*)** for mathematics – fact 6; **(2*)** for mechanics – fact 14, item 4.1.2.1.1.2 (2) and (3); **(3*)** for chemistry – fact 14, item “4.1.2.2.2.3)”.

4) Fourth step: **(1)** as a result of establishment of close links of various forms between the elements, parts and aspects of the given cognitive content **(2.1)** separation of the given scientific knowledge as a self-dependent field of science specified by means of a definite name, and in the same time **(2.2)** formation of specialists, professionals in it **(2.1)→(2.2)** is realized. One may say this step is completion of the first stage of historical process of formation of the given science immediately followed by the next, second stage.

Examples in section 0.2.1: **(1*)** for mathematics – facts 7. Actually the accomplishment of this step in respect of both its place between other steps of formation of the given science and its concrete character of the way of realization usually differ considerably for different sciences. So, **(2*)** in chemistry even after its formal separation from the medicine as self-dependent science a part of the chemists continue to come from the medicine and to be educated in the field of medicine; so taking this fact into account one may say that this step was accomplished in chemistry parallel to some other steps.

5) Fifth step: in the course of further continuation of the process of development of given science a process **(1)** of initial formation of foundational for it ideas and methods of research which secure **(1.1)** its systematisation and at the same time its construction as an unified whole in which various its elements, parts, aspects are internally connected between them in a definite way, and **(1.2)** possibly maximum reliable substantiation of various its parts **(1.1)→(1.2)**

on a general theoretical basis, as well as **(2)** of their philosophical elucidation and substantiation, which can turn out to be debatable problems however with significant importance for further process of formation of the systematic foundations of the given field of science **(1)→(2)** begins.

Examples in section 0.2.1: **(1*)** for mathematics – facts 9, 11, **(2*)** for mechanics – fact 14, item “4.1.2.1.1.2)” (5); **(3*)** for chemistry – fact 14, items “4.1.2.2.2.4)” and “4.1.2.2.2.5)”, **(4*)** for biology – fact 14, item “4.1.2.3.2)”.

6) Sixth step: the processes of formation **(1)** of foundational for the given science ideas and methods of research which secure **(1.1)** its systematisation meanwhile obtaining the form of an unified whole in which various its parts are connected between them in a definite internal way, as well as **(1.2)** the reliable substantiation of its various parts, connected with establishment (explicitly, or at least implicitly) of definite requirements for acceptance of a given cognitive element – concept, problem, statement, etc., in the body (or, more precisely: in the system) of the given science **(1.1)→(1.2)** on a general theoretical basis, and in the same time **(2)** of the given science on a higher level of its theoretical form **(1)→(2)** is accomplished.

Examples in section 0.2.1: **(1*)** for mathematics – fact 12, **(2*)** for mechanics – fact 14, items “4.1.2.1.1.3)” - “4.1.2.1.1.4)”; **(3*)** for thermodynamics – fact 14, item “4.1.2.1.1.2)”, **(4*)** for electrodynamics – fact 14, item “4.1.2.1.3)”, **(5*)** for chemistry – fact 14, item “4.1.2.2.2.6)”, **(6*)** for biology – fact 14, item “4.1.2.3.3)”.

0.2.2.1.3.1.2. After second stage of formation of a special science like mathematics, physics, etc. or of their parts (on the higher level of its theoretical form mentioned above in section 0.2.2.1.3.1.1, item “6)”) its subsequent historical development continues besides the quantitative accumulation of new results is alternated with deep qualitative modification of its form connected with setting new foundations of it.

Examples in section 0.2.1: **(1*)** for mathematics – fact 12, item “4)”; fact 14, item “4.2)”; fact 15, item “2.1)”; fact 16, item “1)”; **(2*)** for theoretical physics – fact 15, item “2.2)”; **(3*)** for biology – fact 15, item 2.3.

0.2.2.1.3.1.3. After first stage of the historical process of formation of special sciences (cf. section 0.2.2.1.3.1.1, item “4)”) sometimes between some of them essential connections of various kinds and strength appear - **(1)** they enter in such a kind of interdisciplinary connections one with another, that one of both sciences turns out to play a role of foundations for the other science, **(2)** they enter into equivalent interdisciplinary connections, **(3)** they possess

common (or almost common) abstract structures in consequence of which similar methods of research take place in them, etc.

Examples: (1*) about the first kind of connections - as a result of formation of physical chemistry between physics and chemistry as well as of biochemistry between chemistry and biology it turns out at least in some cases and degree the biology to find its foundations in the field of chemistry and further the chemistry – in the field of physics, cf. fact 14, item “4.1.2.2.7)”, fact 15, item “2.6.1)”; **(2*)** about the third kind of connections – undulatory theory of light and undulatory theory of sound possess one and the same structure studied in the general wave theory.

0.2.2.1.3.1.4. In consequence of establishment of connections between various sciences, respectively between various scientific theories, after first stage of the historical process of their formation, stronger or weaker, more or less, on a large scale *unification* of them takes place as a manifestation of integrative processes in the history of science in contrast and in the same time as addition to the processes of differentiation in it. The processes of formation and further development of such a kind of unification of sciences and scientific theories also passes on different steps like those mentioned in section 0.2.2.1.3.1.1.

Examples from section 0.2.1: (1) The result of unification of some fields of the science as mechanics, acoustics, optics, electrodynamics, thermodynamics, etc. formed during the period of time 16th–19th century on the basis of establishment of some connections between them of different character is the concept of classical physics – cf. fact 14, item “4.1.2.1)”; **(2)** the result of further unification of physics with chemistry, biology, geology, etc, is the concept “natural science” - cf. fact 14, item “4.1)”; **(3)** the result of further unification of the natural science with mathematics as well as with scientific basis of pragmatic fields of theoretical knowledge as technology, medicine, etc. during the period of time 16th-19th centuries is the concept “classical science” - cf. fact 14, items “1)–“4)”. Other types of unification of elements of different sciences is demonstrated by means **(4)** of formation of physical chemistry and biophysics – cf. fact 14, item “4.1.2.2.2.7)” and fact 15, item “2.6.3)”; **(5)** of formation of cybernetics – cf. fact 15, item “2.6.3)” and fact 16, item “1)”.

0.2.2.1.3.1.ω. At the end of empirical treatment of the problem concerning contentful (full of mater) explication of the concept Sc_{htknl}^{crp} we may state the following:

1) In accordance with the revealing two stages of the historical process of formation of sciences in section 0.2.2.1.3.1.1 we have to make distinction between following two further explications of the concept “science” in the sense of $\mathbf{Sc}_{htknl}^{crp}$:

1.1) *Primary formed (theoretical) science* denoted as “ $\mathbf{Sc}_{prhtknl}^{crp}$ ” – this is such a science $\mathbf{Sc}_{htknl}^{crp}$ which has realised the third step of formation (accordingly to the list of steps in the section 0.2.2.1.3.1.1).

1.2) *Developed (theoretical) science* denoted as “ $\mathbf{Sc}_{dvhtknl}^{crp}$ ” – this is such a science $\mathbf{Sc}_{htknl}^{crp}$ which has realised the sixth step of formation (accordingly to the list of steps in the section 0.2.2.1.3.1.1).

This semantic splitting the concept “science” in the sense $\mathbf{Sc}_{htknl}^{crp}$ into further two explications just mentioned above can be presented by means of disjunctive-concept-vector in the following way:

$$[\mathbf{Sc}_{htknl}^{crp}] = {}_V \begin{bmatrix} \mathbf{Sc}_{prhtknl}^{crp} \\ \mathbf{Sc}_{dvhtknl}^{crp} \end{bmatrix} \quad \{52\}$$

The disjunction is of non-excluding type of the following sense:

$$\mathbf{Sc}_{prhtknl}^{crp} \subset \cdot \mathbf{Sc}_{dvhtknl}^{crp} \quad \{53\}$$

(About the definition of the concept of set-theoretical relation “strict set-inclusion” – cf. {36}).

As a consequence of {53} we have:

$$\text{If } X \text{ is } \mathbf{Sc}_{dvhtknl}^{crp}, \text{ then } X \text{ is } \mathbf{Sc}_{prhtknl}^{crp} \text{ too.} \quad \{54\}$$

$$\text{If } X \text{ is } \mathbf{Sc}_{prhtknl}^{crp}, \text{ then it is possible } X \text{ to be } \mathbf{Sc}_{dvhtknl}^{crp}. \quad \{55\}$$

Note to formulae {54} and {55}. In order to fulfill the requirement of the section 0.1.2 for proposing in these prolegomena an *abridged and simplified* treatment of the present topic a formalised presentation of formulae {54} and {55} will be avoided here.

2) Taking into account in particular (1) the essential role in process of formation of various special sciences of the two mutually opposite tendencies – integrative and differential one, mentioned in section 0.2.1, fact 14, item “3)”, and (2) the role of establishment of essential connections of various kinds and strength between sciences, respectively between scientific theories, after first

stage of their formation, for *unification* of them in various forms, mentioned in sections 0.2.2.1.3.1.3 and 0.2.2.1.3.1.4, above, we have to point out *the hierarchical structure* of the science at whole as well as of various fundamental sciences and scientific theories which can be of qualitatively different character.

Corollary from sections 0.2.2.1.3.1.1 - 0.2.2.1.3.1.ω.

1) Taking into account section 0.2.2.1.3.1.ω, item “1)” and especially in view of {53} (and in the same time of {54} and {55}) the concept of science in the sense of $\text{Sc}_{dvhtknl}^{crp}$ will be subjected to theoretical treatment including definition and its semantic analysis in the next section 0.2.2.1.3.2.

2) Taking into account section 0.2.2.1.3.1.ω, item “2)” in order to give to the concept “science” more precise and in the same time clearer form we have further to make distinction between two basic types of *hierarchical structure* of various fundamental sciences and scientific theories outlined already in section 0.2.2.1.3.1.4 which are named here as “canonical” and “interdisciplinary”, respectively. Correspondingly we obtain two basic types of sciences, respectively of scientific theories, as well as of their classification: (1) canonical ones, and (2) interdisciplinary ones.

Such an explication of concept of science Sc_{htknl}^{crp} connected with distinction between canonical and interdisciplinary types of them can be presented as a disjunctive concept-vector in the following way:

$$[\text{Sc}_{htknl}^{crp}] = \begin{bmatrix} S_{cnhtknl}^{crp} \\ S_{idshtknl}^{crp} \end{bmatrix} \quad \{56\}$$

where (1) the explication of concept of science “ $\text{Sc}_{cnhtknl}^{crp}$ ” means “science in the sense Sc_{htknl}^{crp} of canonical type”, or otherwise speaking – “canonical science Sc_{htknl}^{crp} ”; (2) the explication of concept of science “ $\text{Sc}_{idshtknl}^{crp}$ ” means “science in the sense Sc_{htknl}^{crp} of interdisciplinary type”, or otherwise speaking – “interdisciplinary science Sc_{htknl}^{crp} ”.

In view of the topic of present prolegomena the explication of concept “science” $\text{Sc}_{idshtknl}^{crp}$ is of more special interest and consequently we have to pay it more attention.

**0.2.2.1.3.2. Theoretical treatment of the problem concerning
contentful (full of mater) explication of the concept “science” in the sense of
 $Sc_{dvhtknl}^{crp}$: its definition and semantic analysis**

**0.2.2.1.3.2.1. Formulation of the explication of concept “science” in the sense
of $Sc_{dvhtknl}^{crp}$**

In view (1) of the facts of history of science mentioned in section 0.2.1 and (2) of the results of empirical treatment of the problem concerning contentful (with regard to the contents) explication of the concept Sc_{htknl}^{crp} formulated in section 0.2.2.1.3.1, as well as (3) of some general considerations, (1)→(3) the following explication of the concept “science” $Sc_{dvhtknl}^{crp}$ may be postulated:

Df. $Sc_{dvhtknl}^{crp}$. The concept “science” in the sense of $Sc_{dvhtknl}^{crp}$ means (1) separate, relatively self-dependent and specified by a name, (2) well systematized variety of cognitive elements unified in a totality on the basis of a given foundational kernel, (3) represented in the form of (usually hierarchical) S-models, (4) well-grounded by means of the historically developing (4.1) experience (observations and experiments) and/or (4.2) logical substantiation, (5) knowledge of reality, (6) ultimately destined to satisfy some needs of the human life.

**0.2.2.1.3.2.2. Analysis of the formulation of explication
of concept “science” in the sense of $Sc_{dvhtknl}^{crp}$**

**0.2.2.1.3.2.2.1. Concerning the meaning of item (5) of Df. $Sc_{dvhtknl}^{crp}$:
science as a knowledge of reality**

Characterizing the science in the sense of $Sc_{dvhtknl}^{crp}$ as knowledge of reality the following is presupposed:

1) The concept “knowledge” is the nearest genus of the concept “science”. So, the rest items in the formulation of **Df. $Sc_{dvhtknl}^{crp}$** express various specifications of the concept “science” as a kind of knowledge.

2) The knowledge possesses various forms among which the following two pairs of basic forms will be specially mentioned:

2.1) *descriptive and prescriptive forms of knowledge*. So, the human knowledge is not merely and not only a direct reflection of reality, but it contains also values and prescriptions expressing subjective attitude towards various facts of reality, relevant to the human actions (meanwhile it can turn out

also to be in some way objectively determined and in such a way to be indirect reflection of the reality);

2.2) abstract and concrete forms of knowledge. They are possessed of various specific kinds and they are in mutual connection in the process of knowledge. So, by means of the connection with concrete forms of knowledge which more directly reflect the separate facts of the empirically given reality the abstract forms of knowledge turn out also to have epistemic value in respect to last ones, besides in increasing volume.

**0.2.2.1.3.2.2.2. Concerning the meaning of item (3) of Df. $Sc_{dvhtknl}^{crp}$:
science as a knowledge of reality “represented in the form of
(usually hierarchical) S-models”**

1) Under the concept "S-model" we understand first of all an explication of the intuitive (naive) notion of scientific model usually used as a tool in the scientific investigations and as means of formulation of their results in the form of scientific theories or hypotheses, as well as circulating in the scientific communication.

2) Roughly speaking, in accordance with this explication the concept "S-model" is treated as revealing a relation (named **m**-relation) between two systems, called "system-object" and "system-model", respectively. So, by the term “S-model” (“System-model”) will be denoted the triplet

$$< S_{Obj} , S_{Mdl} , \mathbf{m} > ,$$

where S_{Obj} denotes “object-system”, respectively “the object of the model”, S_{Mdl} denotes “modelling system”, respectively “the model”, m denotes “modelling relation (in direction from the object-system to the modeling system)”.

3) In accordance with the preliminary, very general treatment of conception “S-model” in items “1)” and “2)” above in order to formulate definition of it is necessary previously to propose at least in a very general form of semantic explication of the concept “system” formulating definition of its content.

Preliminary-Df. Syst X We shall say that **X** is a system and shall denote it as “**Syst X**”, if and only if **X** is a non-empty, in the general case many-sorted set Ω , supplied with sets of hierarchical (i.e. in varying degrees) **(1)** predicates **P** (properties and relations), **(2)** functions **F**, and **(3)** operations **O**, closed in Ω :

$$\mathbf{Syst\ X} = < \Omega, \{P_j\}, \{F_k\}, \{O_l\} > , \quad \{57\}$$

where $\Omega \neq \emptyset$ and in the general case $\Omega = \bigcup_i \Omega_i$, $i, j, k, l \in I$, I is a set of indexes in the general case of arbitrary character.

Df. Bs(X), Str(X) The set Ω in equation {54} will be called “the basis of the system X ” and will be denoted as “**Bs(X)**”; the ordered set in it

$$\langle \{P_j\}, \{F_k\}, \{O_l\} \rangle$$

will be called “structure of the system X ” and will be denoted as “**Str(X)**”.

Note to Preliminary-Df. Syst X. Under “hierarchical (i.e. in varying degrees) (1) predicates **P** (properties and relations), (2) functions **F**, and (3) operations **O**” in Preliminary-Df. Syst X roughly speaking we shall understand such kind of predicates, functions, and operations, which are distinguished between different degrees 1,2,3, and so on in such a way that for those of first degree the field of their arguments and values are only elements of the basis of given system Ω ($\Omega = \text{Bs}(X)$); for those of second degree the field of their arguments and values is obtained as a result of enlargement of the field of arguments and values of the predicates, functions, and operations of first degree by means of including to it subsets of Ω , as well as predicates, functions, and operations of the first degree; further for those of third degree the field of their arguments and values is obtained as a result of enlargement of the field of arguments and values of the predicates, functions, and operations of second degree by means of including to it also sets of subsets of Ω , as well as predicates, functions, and operations of the second degree, and so on. If we give to this reasoning a strong form of inductive definition we shall obtain complete definition of the concept “hierarchical (i.e. in varying degrees) (1) predicates **P** (properties and relations), (2) functions **F**, and (3) operations **O**”. Besides we speak about predicates, functions, and operations as about different entities because in addition to their extensional treatment we allowed also their intentional treatment.

3.1) Df. vSyst X We shall say that X is a *very system* (i.e. “that X is a system in the narrow sense”) and shall denote it as “**vSyst X**” if and only if

$$(\Omega \neq \emptyset) \ \& \ ((\{P_j\} \neq \emptyset) \vee (\{F_k\} \neq \emptyset) \vee (\{O_l\} \neq \emptyset)). \quad \{58\}$$

3.2) Df. sSyst X We shall say that X is a *semi-system* (i.e. “that X is a system in the non-proper sense”) and shall denote it as “**sSyst X**” if and only if

$$(\Omega \neq \emptyset) \ \& \ (\{P_j\} = \emptyset) \ \& \ (\{F_k\} = \emptyset) \ \& \ (\{O_l\} = \emptyset). \quad \{59\}$$

Th. It is obvious that the concepts “semi elementary system” and “non-empty set” are extensionally equivalent, but intentionally different:

$$X \text{ is sSyst } X \text{ if and only if } ((X = \Omega) \ \& \ (\Omega \neq \emptyset)) \quad \{60\}$$

Note to items “3.1)” and “3.2)” of section 0.2.2.1.3.2.2.2. We treat here the concept of elementary system in a very general way including in it as a partial case merely the concept of non-empty set on the grounds similar to those in the probability theory allowing us to treat the improbable event as possessing probability which is equal to zero, as well as in mechanics

allowing us to treat the body without motion as possessing definite velocity of movement which namely is equal to zero.

4) Df. S-Mdrel m We shall say that *there is a relation of S-modelling of the system X into the system Y* and will write

$$\mathbf{XmY}$$

if and only if there is at least partial and approximate homomorphism of the system X into the system Y.

In such a case the system X plays the role of an object-system and the system Y - the role of modelling-system, i.e. $S_{Obj} = X$, and $S_{Mdl} = Y$.

Note to Df. S-Mdrel m. Roughly speaking by the term “partial and approximate homomorphism of the system X into the system Y” just used above it is denoted such a kind of relation between systems X and Y as the correspondences of the system X into the system Y which are characteristic for the homomorphism of X into Y in the usual sense, but they are valid at least for some elements of the system X as well as at least for some components of its structure $\langle \{P\}, \{F\}, \{O\} \rangle$, besides, at least in an approximate way.

Df. S-Mdl The system Y is S-model of the system X if and only if between systems X and Y there is S-modelling relation:

$$\mathbf{XmY}.$$

Df. pure S-Mdl The subsystem Y' of the system Y, $Y' \subseteq Y$, will be called *pure S-model of the system X* (resp. proper modelling-system in respect to the system X) if and only if it contains only those elements, predicates, functions, operations of Y, which are put in correspondence with elements, predicates, functions, operations of X, respectively, in the homomorphism $\mathbf{XmY}$.

Df. S-Mdg We shall call *process of S-modelling* the totality of the processes of (1) construction, (2) investigation and (3) application of S-models of a given subject of knowledge.

Th. of variety of S-Mdl and S-Mdg (1) Depending on the variety of concrete forms of the (1.1) object-systems, (1.2) modelling systems and (1.3) m-modelling relations (1.1 $\Rightarrow$ 1.3) (2) there is a great variety of corresponding forms of S-models; besides also (3) depending in addition on the variety of concrete forms of the (3.1) formation, (3.2) investigation and (3.3) application (3.1 $\Rightarrow$ 3.3) of S-models (4) there is a great variety of corresponding forms of the processes of S-modelling.

Note to thesis of variety of forms of S-models and of S-modelling. The thesis of variety of S-models and of S-modelling is specially formulated above in order to resist the wide spread tendency of specialists of various special sciences speaking about methodological problems of scientific modelling to take in consideration only very limited

class of kinds of models as well as of forms of procedure of the scientific modeling which are usually used and well known in their proper field of investigation ignoring in the same time sheer vastness of kinds of models and of forms of procedure of the scientific modeling which are used in the other branches of the science. Such a way of treating of the general methodological problems of scientific modeling results into formulation of one-sided conclusions.

OntTh. of S-Mdrel and S-Mdg (1) The relations of S-modelling between some natural systems by means of other natural systems, as well as **(2)** the processes of S-modelling in the form of natural process of formation of model relations between natural systems **(1 $\Rightarrow$ 2)** **(3)** take place among various systems in reality and **(4)** play an essential role in the definiteness of the reality.

5) Th. Sc_{htknl}^{crp} as S-Mdl Every scientific theory developed in a sufficient degree is in essence (nevertheless in explicit or at least in implicit form) **(1)** a kind of S-model of its subject of research **(2)** involving **(2.1)** definitions of concepts, **(2.2)** formulations of problems, **(2.3)** theses (in the forms of theorems, scientific laws, etc.) as solutions of corresponding problems, **(2.4)** prescriptions (especially in the forms of rules), **(2.5)** arguments (in particular, strong logical proofs of theorems, theoretical and empirical demonstrations of laws, etc.), and **(2.6)** methods for posing and solution of scientific problems, respectively, tasks, so that **(3)** the cognitive elements mentioned in item **(2)** **(3.1)** are connected between themselves in such a way that they are unified in systematic theoretical wholes and **(3.2)** obtain their substantiation **(4)** by means of given foundational kernel (cf. below the section 0.2.2.1.3.2.2.4).

Th. of variety of S-Mdls as Sc_{htknl}^{crp} A great diversity of forms of S-models as representatives of sciences or scientific theories in the sense of Sc_{htknl}^{crp} (in particular including descriptive and prescriptive, abstract and concrete forms of S-models - in accordance with that what was mentioned in section 0.2.2.1.3.2.2.1, item “2)”, besides realising various cognitive functions, take participation in the scientific investigations as well as in exposition of their results in the form of theories or hypothesis, so that namely application of appropriate combination of different forms of S-models reinforces considerably the scientific knowledge, increases its quality.

Th. Sc_{htknl}^{prc} as S-Mdg The processes of development of science, otherwise speaking, the science treated as developing theoretical knowledge in the sense of Sc_{htknl}^{prc} , can be treated as a process of constructions, investigations and applications of S-models, denoted as **S-Mdg**.

Th. of variety of S-Mdg in Sc_{htknl}^{prc} In the processes of development of science, otherwise speaking, treating science as developing theoretical

knowledge, one may say various forms of S-modelling can take (and moreover, in many cases it turns out to be reasonable to take) place in it depending on (1) diversity of the forms of used S-models (in accordance with Th. of variety of S-Mdls as Sc_{htknl}^{crp}) and (2) diversity of the ways of application of S-models.

Note to theses of variety of forms of S-models and of S-modelling in the scientific knowledge. The motives of explicit formulation of both theses above - Th. of variety of S-Mdls as Sc_{htknl}^{crp} and Th. of variety of S-Mdg in Sc_{htknl}^{prc} , are the same as those formulated above in the “Note to thesis of variety of forms of S-models and of S-modelling”.

Corollary from Th. Sc_{htknl}^{crp} as S-Mdl (1) The truth and falsehood of a given statement in the given scientific theory *treated in itself* is of analytical character in the framework of the corresponding S-model; so (2) we may not affirm for a given statement accepted (eventually logically derived) in a given scientific system *treated in itself* to give any real information concerning the subject of this theoretical system, respectively concerning the field of objective reality which is studied by last one. However (3) if it is demonstrated that the S-model representing the given scientific system is correct in respect to its subject (i.e. that the m-modelling relation directed from the subject of the theory, i.e. from the object-system, to the given modelling-system representing the given scientific theory really takes a place), then the accepted statements in it (in particular the statements theoretically derived in it in a pure rational way) obtain the value of expressing objective truth, otherwise speaking they obtain the function to give information about the reality. So, the cognitive function of accepted theoretical statements, in particular of the theoretical laws of nature, can be realized only through correspondence of the theory as a whole to its subject, otherwise speaking the accepted theoretical statements, in particular the theoretical laws of nature, correlate the reality through the correlation of the corresponding theoretical S-model to the reality. Besides (4) in the S-models and in the same time in the human knowledge, in particular in the scientific knowledge (treated as part and form of the human knowledge), objective value can possess the structure and in the same time the relations forming the given structure, but not the nature of initial elements of the corresponding object-systems (in the sense of section 0.2.2.1.3.2.2.2, item “4”) of the given S-model.

Corollary from Th. Sc_{htknl}^{crp} as S-Mdl and from Th. Sc_{htknl}^{prc} as S-Mdg The epistemology in the philosophy of science treated as philosophical theory of scientific knowledge can be regarded as a partial case of the more general and on a larger scale all-embracing *theory of S-models and S-modelling*.

Meth. of S-modelling In the course of study of problems concerning systems, S-models and processes of S-modelling it is significant (1) to take

into account their great variety of forms, and (2) to combine their general treatment on the very abstract level with the treatment of their varied concrete forms.

6) About argumentation of the theses formulated above in items “4)” and “5)”: These theses are accepted here as fundamental empirical statements on the basis of the total experience of the humanity in the course of historical of scientific knowledge.

Note to items “4)” and “5)”. The theses formulated above in the items “4)” and “5)” of the section 0.2.2.1.3.2.2.2 can be treated as acceptable in the philosophy of science only in such a degree in which it is verified by means of experience of the scientific investigations that Df. S-model is a good explication of the intuitive (naive) notion of scientific model. At the moment the problem of such verification is still open.

Note to section 0.2.2.1.3.2.2.2. The question about usually hierarchical form of the S-models as representing sciences and scientific theories, mentioned in item 3 of the formulation of explication of concept “science” in section 0.2.2.1.3.2.1, is closely connected with the question about the science as unified totality of variety of cognitive elements well systematized on the basis of a given foundational kernel, mentioned in item 2 of the same formulation. Therefore its treatment will be postponed to be realized in section 0.2.2.1.3.2.2.4.

0.2.2.1.3.2.2.3. Concerning the meaning of item (4) of Df. $Sc_{dvhtknl}^{crp}$: science as a form of knowledge of reality well-grounded by means of experience and logical substantiation

1) A basic distinguishing feature of the scientific statements (i.e. of the statements accepted in a given field of science or in a given scientific theory as true) is to be well argued by means of experience and logic, satisfying also some methodological requirements specific for the various epochs and fields of science, in some cases also for different scientific schools in one and the same field of science and epoch (for example in mathematics during 20th century there are different requirements to the proof of theorems in the school of intuitionism, on the one hand, and in the school of formalism, on the other hand).

Note to item “1)” of section 0.2.2.1.3.2.2.3. (1) By means of such a formulation I don't exclude at all the role of intuition in the process of development of scientific knowledge, but I treat it to be only of heuristic significance, not of demonstrative one. (2) As the history of science demonstrates the basic principle factors for argumentation of the scientific theories are also basic principal factors for generation of the content of scientific knowledge, on the first place for generation of scientific statements - of scientific principles, theorems, scientific

laws, etc. (This assertion concerns namely the scientific knowledge, not the speculative philosophical systems).

2) We can establish the following principal distinction between the role of both basic forms of generation and argumentation of scientific knowledge mentioned above: (2.1) its logical generation and argumentation by means of systematic application of logical inference is an *internal factor* for its elaboration, while (2.2) the experience (s and experiments) is *external factor* for the generation of scientific knowledge as well as in the same time for argumentation of its objective value and meaning.

3) The strict argumentation of scientific statements in its variety of forms - pure logical proofs of mathematical theorems, experimental demonstrations of physical laws, etc., requires the use of exact and sufficiently rich and flexible language for their formulation – *the specific language of science*. So, the last one is distinguished itself by definite syntax and semantics, which allows to make generalization and idealization of the empirically given separate facts. Usually the fulfillment of such a requirement concerning the language of science is connected with construction of appropriate S-models of the subject of scientific research.

Note to section 0.2.2.1.3.2.2.3. According to items “1)” and “3)” in the present section 0.2.2.1.3.2.2.3 the cases when the scientific investigations are exposed by means of use of colloquial expressions, it is not so important *what is said*, but the most significant is *how it is said* – what kind of explications of the basic concepts (correspondingly, of the sense of basic terms) are assumed and how the corresponding statements are argued. It is especially important to have in mind this thesis in the case of treatment of philosophical statements concerning scientific knowledge expressed by means of colloquial expressions.

In this connection we have to stress that the reasoning in the field of philosophy of science, which by definition has to apply scientific way of research, has to bear a resemblance sooner to mathematical texts than to a narratives - usually it has to take a shape of long chain of definitions and logical conclusions at the first glance looking to be aside from the topic of given consideration and only at the end of it a text directly concerning the given topic to be obtained, like the mathematical proof which only after a long chain of successive deductions ends in relatively short formulation of the theorem as a logical conclusion.

0.2.2.1.3.2.2.4. Concerning the meaning of item (2) of Df. $Sc_{dvhtknl}^{crp}$: science as well systematized variety of cognitive elements unified in a totality on the basis of a given foundational kernel

Continuing the treatment of science in the sense of $Sc_{dvhtknl}^{crp}$ as hierarchical S-model in section 0.2.2.1.3.2.2.2 we can formulate here the following statement:

A given field of science or a given fundamental scientific theory (as well as the science in its integrity) in the sense of $Sc_{dvhtknl}^{crp}$ (so, in its well-developed theoretical form) is (1) a complicated hierarchical partially approximate S-model of a part or of an aspect of the system of reality being its subject of research, (2) constructed by means of starting from elementary (i.e. of degree 1) theoretical units and step by step unifying various theoretical units to increasing degree as its components into well systematized whole (3) on the basis of a given foundational kernel consisting of the fundamental factors producing the unity and substantiation of given scientific content as (3.1) relations of cognitive determination – (3.1.1) S-modeling relations, (3.1.2) logical consequences, (3.1.3) directive methodological prescriptions inherent in the system of categories constituting semantic kernel of the language of science, and/or (3.2) of causal relationship in the corresponding subject of research, producing the objective unity in the enormous variety of different phenomena.

Note to the explication of the phrase “science in the sense of $Sc_{dvhtknl}^{crp}$ as hierarchical S-model”. Under the concept “hierarchical S-model” the author of present text has in mind such a kind of complex S-model which is constructed in the following hierarchical way: from the start most elementary S-models are constructed and they play the role of S-models of first degree, afterwards from the last ones more complicated S-models of second order are constructed and so on.

It is very important to be stressed that various S-models of lower degree playing the role of components of the given hierarchical S-model representing a given field of science or a given fundamental scientific theory may diverge considerably in their informational functions, so that discrepancy and even contradictions between them are possible: (1) various components of a given hierarchical S-model can correspond to quite different parts or aspects of its total subject proposing in such a way quite different descriptions of the studied phenomena as well as (2) they can be of quite different categories – together with *adequate S-models* in respect to the subject of investigation reflecting its nature in a considerable degree also *non-adequate S-models*, containing elements which are in discrepancy with the reality, but nevertheless turning out to be convenient for reasoning and calculations in some limited space are allowed in the science, i.e. the last ones can take place as *conditional S-models* of the given subject. It can be indicated to thermodynamics as a typical example of such a kind of hierarchical S-model – the phenomenological thermodynamics and the statistical thermodynamics propose quite different description of one and the same real phenomena in essence, besides the statistical S-model of the heat-phenomena is more adequate and fundamental than the phenomenological S-model, so that the last one can be derived from the first one as approximation but in the same time the S-model representing the phenomenological thermodynamics turn out to be very convenient for treating the problems of engineering. Moreover, even after rejection of the caloric theory of heat during about the middle of 19th century as being at variance with empirical facts the caloric conception remain valid as conditional S-model of heat phenomena which distinguishes as very convenient for considerations in a given limited field of questions owing to some simple mechanical analogies – indeed namely this circumstance explains the choice and successful application of

caloric S-model of heat-phenomena (actually as conditionally one) in his historically foundational work in the field of thermodynamics (cf. section 0.2.1, fact 14, item 4.1.2.1.2; cf. [11, pp. 41-43], [37]).

Such a treatment of S-models connected with the distinction between adequate and conditional ones turns out to be very fruitful for application to various problems concerning human knowledge, in particular **(1*)** concerning the dynamic of S-models in the history of science, **(2*)** concerning derivation of the principle of compatibility between different and even opposite philosophical doctrines treated as conditional S-models, and so on.

Corollary from the explication of the sense of $Sc_{dvhtknl}^{crp}$ as hierarchical S-model. The processes of development of science in its highly theoretical form and in their both ways – historical and individual, in essence are very complicated, hierarchical processes of S-modelling of reality, involving as partial cases internal processes (in respect to the science) of S-modelling of various theoretical units.

Ep-MthTh. The systematic organization of various elements of scientific knowledge in the frameworks of scientific theories is essential from **(1)** epistemological and **(2)** methodological point of view:

1) It is essential from epistemological point of view at least in following three respects:

1.1) For the human rational subject of knowledge (i.e. for the human subject of knowledge being able for rational thinking) the good systematic organization of given science or scientific theory is necessary and sufficient condition for its possessing in essence merely by means of knowing only its foundational kernel – this cognitive situation is a peculiar case of realization of the Mach's principle of economy of thought. It is so because starting from the foundational kernel of the given science or scientific theory the human rational subject of knowledge is able in principle (accordingly to the meaning of the concept “human rational subject of knowledge” mentioned above parenthetically) to derive (or to construct) from this kernel by means of using logical conclusions (and possibly by the help of reference books) the whole detailed content of the given science or scientific theory.

1.2) In the case of correct scientific theories the systematic organization of their elements corresponds to the systematic organization of the elements of that fragment of reality which is subjected to study, i.e. in such a case the systematic form of scientific theories like their content also is reflection of the real systems and in such a way it turns out also to possess objective cognitive value.

1.3) Making clear the connections between various elements of scientific theories the good systematic form of scientific theories attach a visible character to them.

2) It is essential from methodological point of view at least in following two respects:

2.1) By virtue of its epistemological significance mentioned above in item “1)”, the systematic form of scientific theories as well as of the science as a whole plays a significant heuristic-methodological role to raise in a systematic way problems for research and in such a way to increase the degree of completeness of the scientific knowledge.

2.2) By virtue of its epistemological significance mentioned above in item “1)”, as well as of its methodological significance just mentioned in item “2.1)”, the type of systematization of various elements of scientific knowledge is significant for the way of development of science at the given historical epoch and as a consequence of this circumstance an essential qualitative change in the type of systematization of elements of scientific knowledge in the course of evolution of science determines its transition to a new stage or even to a new period of historical development. Consequently, *analysis of the type or types of systematization of elements of scientific knowledge as well as of the tendencies of its or their change during a given (contemporary for the researcher) epoch is of fundamental importance for elucidation of the place of this epoch in the chain of successive separate links expressing different periods and stages of the historical process of development of science and on such a concrete historic-epistemological basis the philosopher of science to achieve a solution of the actual methodological problems about objectively well-grounded strategic tasks of the further development of science.*

Note to explication of the phrase concerning foundational kernel in item (3). The text of item “(3)” in the basic formulation at the beginning of present section 0.2.2.1.3.2.2.4 can not be treated as containing a definition of the concept “foundational kernel” because the adverb “as” in it expresses an essential moment of indefiniteness. Consequently the last one, i.e. the adverb “as” in the given case (as well as the item “(3)” in its integrity) obtains two kinds of interpretation: **(1)** as indication (expression) of a scheme of definition from which an exact definition is obtainable by means of replacement of as-expression by exact indication of the fundamental factors producing the unity and substantiation of given scientific content, or **(2)** as indication (expression) of a hypothetical definition which is tentatively accepted and applied with readiness for both to be ultimately accepted or to be correspondingly changed as result of its verification (resulting its tentative application).

In the case of following considerations namely the second interpretation will be accepted and applied.

**0.2.2.1.3.2.2.5. Concerning the meaning of item (6) of Df. Sc^{crp}_{dvhtknl} :
science as a kind of knowledge ultimately destined to satisfy some needs of
the human life**

Speaking about science as a kind of knowledge ultimately destined to satisfy some needs of the human life the author of present text has in mind first and foremost satisfying by the scientific knowledge the following two kinds of needs: **(1)** pragmatic needs of the human society, and **(2)** intellectual needs of the human individuals.

1) Speaking about satisfying *pragmatic needs* of the human society by the scientific knowledge I have in mind its potentiality to be applied into social practice (in a broad sense including also individual actions) in its various forms - the processes of production of material wealth, the medicine, the education, etc. Namely this pragmatic function of science, which realization depends from the level of its development on the one hand and from the concrete social order on the other hand is the deep reason for the social stimulus for development of science.

2) Speaking about satisfying *intellectual needs* of the human individuals by the scientific knowledge I have in mind its potentiality to meet intellectual requirements of the human beings as ones of satisfying curiosity, of demonstration of personal intellectual ability, of feeling pleasure from the creative intellectual activity possessing a considerable social value, etc. It is quite clear that the realization of such a function of scientific knowledge is essential reason many persons to devote their life to science, besides namely this devotion plays a considerable role for the progress of science.

Note to section 0.2.2.1.3.2.2.5. There is mutual connection between these two functions of the science, so that they interlace in the process of its development.

**0.2.2.1.3.2.2.6. Concerning the meaning of item (1) of Df. Sc^{crp}_{dvhtknl} :
science as separate, relatively self-dependent and specified by a name
knowledge**

I suppose that **(1)** the meaning of the phrase “science as separate, relatively self-dependent and specified by a name knowledge” is sufficiently clear for correct methodological applications, as well as **(2)** the truth of corresponding statement expressing the attachment of such a feature to sciences and to scientific theories to be undoubtedly. Moreover, it seems this statement to be quite trivial and namely this circumstance explains the fact that usually this distinguishing feature is omitted in the explicit characteristic of the science. So, in the light of all these facts the author of present text has to explain why this trivial feature of the science is explicitly formulated here.

The reason concerning the author of present investigation to do so consists in the following two facts taken together:

1) Following strictly the empirical way of treatment of the given topic the author of present investigation makes every effort to derive the characteristic of science from the empirical facts of the history of science concerning the process of formation of fundamental sciences;

2) In accordance with the previous item “1)” we may state that the empirical facts of the history of science show in the initial, spontaneous process of natural formation of a science usually the very first historical elements of its content grow up on a non-proper soil – in the framework of another science or even of another (non-scientific) form of the knowledge and only after a sufficient degree of development of this content (not only of quantitative accumulation of new elements of knowledge, but also of the qualitative improvement of its form by means of establishment of connections between its elements) the very important step of its separation and specification by means of supplying a name of a new science take place – cf. section 0.2.2.1.3.2.2.6, fourth step.

Corollary from section 0.2.2.1.3.2.2.6. In the course of investigations in the field of history of a given science we have to pay attention to the process of its formation connected with the step of its separation and its specification by means of giving to it a new name denoting a new science. In the same time we have to take into account that the realisation of this step is only the first stage of the process of formation of the given science after which follows the next, second stage of its final formation (cf. 0.2.2.1.3.1.1, items 5 and 6).

Corollary from section 0.2.2.1.3. After obtaining the given above explication of content of the concept (respectively, of the sense of the term) “science” in the present section 0.2.2.1.3 in accordance with the requirements of the section 0.2.0, thesis 1, item 1, we may and even have to pass to making explication of content of the next, second concept mentioned in the last one, namely of the concept “foundations of science”.

0.2.2.2. Explication of the concept “foundations of science”

0.2.2.2.0. Preliminary notes concerning the way of explication of the concept “foundations of science”

1) Introducing the concept “foundations of science” in the present discourse we have first of all to clarify its content, otherwise speaking we have to formulate an answer to the question “What we should understand under the term “foundations of science”?”.

2) Taking into account the actual conditions of its introducing in the present investigation according to sections 0.2.1 and 0.2.2.1, in particular we have to take into account in this connection the following three historic-scientific facts:

2.1) On the first place we have to take into account in this connection the fact that in the course of historical development of mathematics during 19th century and especially during the first half of 20th century **(1)** a peculiar type of investigations in the various branches of mathematics called “foundational” was formed and **(2)** later as a result of its enlargement and generalisation the concept “foundations of mathematics” was formed. Actually this concept was without strictly definite content owing (at least partially) to the fact that it obtained different concretisation in the case of different directions – logicism, intuitionism, formalism, etc. So, generally speaking the term “foundations of mathematics” denotes rather a conceptual scheme, than a strictly defined concept (cf. section 0.2.1, fact 14, item “4.2.2.3.3”, fact 15, item “1”, fact 16, items “1)” and “2,1)”), however it is transmutes into denotation of a definite concept in the cases when it is used in the framework of a given direction, respectively school, as logicism, intuitionism, formalism, etc.,

Besides taking also into account the fact that the mathematics is a kind of science we have to require correspondingly the concept “foundations of mathematics” to turn out to be a kind of the concept “foundations of science”, or, otherwise speaking, we have to require the concept “foundations of science” to be generalization of the concept “foundations of mathematics”.

2.2) On the second place we have to take into account in this connection that in the section 0.2.2.1.3.1.1 we have stated as the final, sixth, step of the canonical process of formation of a given science or fundamental scientific theory the processes of formation **(1)** of foundational for the given science ideas and methods of research which secure **(1.1)** its systematisation meanwhile obtaining the form of an unified whole in which various its parts are connected between them in a definite internal way, as well as **(1.2)** the reliable substantiation of its various parts.

2.3) On the third place we have to take into account that in the course of semantic analysis of concept “science” in section 0.2.2.1.3.2.1, based on the historic-empirical facts listed in section 0.2.1, we have stated as a characteristic feature of the science in particular that it is constructed from elementary (i.e. of degree 1) theoretically units which are unified step by step into various theoretical wholes of increasing degrees by means of given foundational kernel. In section 0.2.2.1.3.2.2.4 it was revealed that the foundational kernels can obtain different forms in the cases of different sciences and fundamental scientific

theories, besides generally speaking they are consisting of the fundamental factors producing the unity and substantiation of given scientific content as (1) relations of cognitive determination – (1.1) S-modeling relations, (1.2) logical consequences, (1.3) directive methodological prescriptions inherent in the system of categories constituting the semantic kernel of the language of science, and/or (2) of causal relationship in the corresponding subject of research, producing the objective unity in the enormous variety of different phenomena.

3) The comparison of what is formulated in item “2.3)” with that what is formulated in items “2.1)” and “2.2)” leads to some consequences relevant to the question “What we should understand under the term “foundations of science”?”:

3.1) As a result of comparison between sections 0.2.2.1.3.1.1 and 0.2.2.1.3.2.2.4 we arrive at the conclusion that the concept of foundational kernel of a science, which is analysed semantically in the section 0.2.2.1.3.2.2.4, plays the role of explication of the concept of “foundational for the given science ideas and methods of research” mentioned in section 0.2.2.1.3.1.1 as the characteristic of the final (sixth) step of the canonical process of formation of a given science or fundamental scientific theory. In consequence of this connection we can formulate: The final (sixth) step of the canonical process of formation of a given science or fundamental scientific theory consists in elaboration of its foundational kernel.

3.2) Revealing the connection between foundational studies in the two different levels of mathematics mentioned above in item “2.1)” and the concept of foundational kernel (of a science or scientific theory) we can formulate: (1) foundational studies in various branches of mathematics as set theory, theory of natural numbers, geometry, algebra, probability theory consist in elaboration of their foundational kernel which in these cases takes a form of elaboration of appropriate axiomatic system; (2) various directions in foundations of mathematics (as a whole) as logicism, intuitionism, formalism, etc. in essence present various programmes for elaboration of general foundational kernel of mathematics (as a whole).

0.2.2.2.1. Formulation of the explication of concept “foundations of science”

1) In view of the conditions of introducing the concept “foundations of science” in the present discourse, formulated above in section 0.2.2.2.0, the following its explication is postulated here:

The foundations of a given science, of the science as a whole or of fundamental scientific theory are (1) complete systematic theoretical exposition (representation) of its foundational kernel (in the sense expressed in section 0.2.2.1.3.2.2.4) together with (2) demonstration of the role of this foundational kernel for its systematic construction by means of theoretical derivation (from this foundational kernel) of the most characteristic as well as of the most important (in a figurative sense: of the key) propositions of its content.

2) In accordance with the formulation of explication of the concept “foundations of science” above in item “1)” to be established the foundations of a given science or scientific theory means to be realised the following two steps:

2.1) first of all to be formulated its foundational kernel in a complete systematic theoretical form, and

2.2) on the second place to be demonstrated that starting from the given theoretical formulation of the foundational kernel and carrying out the corresponding procedures according to it we can obtain the multiformity of propositions making up the content of the given science or scientific theory (usually it is enough to obtain the smaller set of its most important and characteristic propositions).

Note to 0.2.2.2.1, item “2.2)”. (1) The fulfilment of requirements of items “2.1)” and “2.2)” immediately above is treated here from pure special theoretical point of view as sufficient for establishment of foundations of a given science or scientific theory. Simultaneously it is worthy to be stressed that from methodological point of view the proposed foundations of a given science or scientific theory obtain more clear and in the same time more convincing character if this pure special theoretical elaboration of foundations of a given science or scientific theory consisting in pure theoretical treatment of its foundational kernel in a way as described immediately above in items “2.1)” and “2.2)” is supplemented with elucidation of the historical process of formation and development of its foundational kernel in the framework of the history of science.

(2) From the other side the historical treatment of process of formation of the foundational kernel of given science or scientific theory in the framework of history of science obtains a logical substantiations and in the same time more convincing form if it is supplemented with pure theoretical treatment of the foundational kernel in a way as described immediately above in items “2.1)” and “2.2)”.

(3) The statements formulated in items (1) and (2) above taken together demonstrate the close mutual connection between investigations in the field of foundations of science, in particular in the field of philosophy of science, and in the field of history of science.

Note to 0.2.2.2.1. Methodological thesis: The foundational kernel of a given science or scientific theory is to its foundations as the essence to its concrete realisation.

Taking into account the general statement that a given essence in principal may be realised in different ways we may accept that a given foundational kernel of a given science or scientific theory in principal can be realised in the form of foundations of the given science or scientific theory, respectively, in different ways.

0.2.2.2.2. Argumentation of the explication of concept “foundations of science” formulated above.

The formulation of the explication of concept “foundations of science” in the section 0.2.2.2.1 conforms to conditions of introducing in the present discourse the concept “foundations of science” formulated in section 0.2.2.2.0, item “2)”.

1) In accordance with the mentioned in section 0.2.2.2.0, item “3.2)” the formulation of explication of concept “foundations of science” in the section 0.2.2.2.1 conforms to the conditions of introducing the concept “foundations of mathematics” as well as the conceptual scheme “foundation of a given mathematical branch” formulated in section 0.2.2.2.0, item “2.1)”:

1.1) Various systems of foundations of mathematical theories as set theory, theory of natural numbers, algebra, geometry, probability theory, by means of postulation of appropriate axiomatic system realised in the history of mathematics, can be treated as theoretical descriptions **(1.1)** of the content of a given foundational kernel in the form of axiomatic systems, and afterwards **(1.2)** of the work of this kernel producing various details of the given mathematical theory by means of logical derivation ultimately from the axioms of a variety of propositions constituting the content of the given mathematical theory. For example, the axiomatic systems formulated in Euclid’s “Elements” (cf. section 0.2.1, fact 12), in Peano’s “Arithmetices principia, novo methoda exposita” (“The principles of arithmetic, presented by a new method” – cf. section 0.2.1, fact 15, item “1.1.2)”), in Hilbert’s “Grundlagen der Geometrie” (Foundations of geometry” – cf. section 0.2.1, fact 15, item “1.3)”), Kolmogoroff’s “Grundbegriffe der Wahrscheinlichkeitsrechnung” (cf. section 0.2.1, fact 16, item “2.1.5.2)”) can be treated in such a way.

1.2) Various directions in the foundations of mathematics (as a whole) during about the first half of 20th century as logicism, intuitionism, formalism, etc., in essence proposed corresponding types of foundational kernel for mathematics, besides the distinction between them consists in the distinction between accepted in them types of foundational kernel (cf. section 0.2.1, fact 15, items “1)” and “3)”, fact 16, item “2.1)”).

2) In accordance with the mentioned in section 0.2.2.2.0, item “3.1)” the formulation of the explication of concept “foundations of science” in the section 0.2.2.2.1 conforms to second and third conditions of introducing the concept “foundations of science” concerning the natural sciences mentioned in section 0.2.2.2.0, items “2.2)” and “2.3)”, taken together.

Actually the role of foundational kernel for formation of chemistry as a science played the periodic law embodied in the periodic table of chemical elements (cf. section 0.2.1, fact 14, item “4.1.2.2.2.6”), also item “4.1.2.2.2.7”) and for formation of biology as a science – Darwin’s theory of evolution (cf. section 0.2.1, fact 14, item “4.1.2.3.3.5”) – in the both cases namely “causal relationship in the corresponding subject of research, producing the objective unity in the enormous variety of different phenomena”, which in accordance with item “(3.2)” of the basic text of section 0.2.2.1.3.2.2.4 is a form (respectively, element) of the concept “foundational kernel”, takes place.

3) The comparison between foundational kernels for mathematics mentioned in item “1.1)” above and for natural sciences mentioned in item “2)” above demonstrates that they are different; consequently, the concept “foundational kernel of science” is more general than the concept “foundational kernel of mathematics” and correspondingly the concept “foundations of science” is more general than the concept “foundations of mathematics” so, the requirement formulated at the end of item “2.1)” of the section 0.2.2.2.0 is fulfilled.

Note to section 0.2.2.2.2. The inclusion of elements of the philosophy of science in the foundations of science also is demonstrated in the history of science on a more concrete level. A sharp demonstration of such kind of inclusion takes place during 20th century in the denotation of directions in both foundations of mathematics and philosophy of mathematics by means of one and the same term: the broadly accepted terms “logicism”, “intuitionism”, “formalism” denote in the same time a given direction in the philosophy of mathematics as well as corresponding directions in the foundations of mathematics.

0.2.2.3. On explication of the concept “interdisciplinary approach to foundations of science”

0.2.2.3.1. Towards the preliminary elucidation of meaning of the concept “interdisciplinary approach to foundations of science”

0.2.2.3.1.0. Notes concerning the way of preliminary elucidation of the meaning of concept “interdisciplinary approach to foundations of science”

After explication of the concepts “science” in section 0.2.2.1 and “foundations of science” in section 0.2.2.2 it remains for the preliminary elucidation of the meaning of the concept “interdisciplinary approach to foundations of science” to be preliminary elucidated the meaning of concepts “approach (to a scientific problem)” and “interdisciplinary approach (to a scientific problem)”. Consequently, we have to realise following three steps:

- 1) Step one: preliminary elucidation of the meaning of concept “approach (to a scientific problem)”;
- 2) Step two: preliminary elucidation of the meaning of concept “interdisciplinary approach (to a scientific problem)”;
- 3) Step three: preliminary elucidation of the meaning of concept “interdisciplinary approach to foundations of science”.

0.2.2.3.1.1. Preliminary elucidation of the meaning of concept “approach (to a scientific problem)”

0.2.2.3.1.1.1. The general intuitive idea of the concept “approach (to a scientific problem)” can be roughly expressed with other words as it follows: “the way of starting the research (of the given scientific problem) which is reflecting in the course of action in the next successive steps of the given research”. We can set its explication by means of analysis of its structure.

The approach to a scientific problem consists first of all of following two types of components: **(1)** formal components – these are the general forms of the way of starting the research (of the given scientific problem) and **(2)** conceptual components (in German: inhaltliche Bestandteile) – these are various characteristic (“key”) elements of the research – significant concepts, questions, theses, methods, etc. used in the given research which in their totality express its peculiarity.

Note to section 0.2.2.3.1.1.1. For example for Cantor’s set theory the formal component of the approach to investigation of its various problems can be determined as abstract conceptual one and among its conceptual components is the concept “actual infinity”; the formal component of the Hilbert’s approach to foundations of geometry is the axiomatic method in an very abstract and analytic (in view of separating all accepted axioms into five groups) form, besides its conceptual component consists of the choice of the three basic elements of geometry and the abstract relations between them fixed at the very outset of formulation of the axiomatic system of geometry as well as of the choice of accepted in it axioms (cf. [39]); the formal component of the approach to construction of the phenomenological thermodynamics is the method of phenomenological principles in a definite mathematical form besides among its conceptual components are the concepts “energy” and “entropy” as well as the accepted principles which are subjected to a process of historical development – initially only two principles were accepted as foundation of thermodynamics, recently four principles are accepted (this fact expresses the historical development of the given approach).

0.2.2.3.1.1.2. Between some mutually opposite approaches there is a relation of compatibility which allows in such kind of cases their combined application in a given investigation.

0.2.2.3.1.2. Preliminary elucidation of the meaning of concept “interdisciplinary approach (to a scientific problem)”

The preliminary elucidation of meaning of the concept “interdisciplinary approach (to a scientific problem)” presupposes preliminary elucidation of the meaning of concept “interdisciplinarity (in science)”, correspondingly of concept “interdisciplinary science” (as well as “interdisciplinary theory”). The last one was already introduced in essence in the present discourse in sections 0.2.2.1.3.1.3, 0.2.2.1.3.1.4, 0.2.2.1.3.1.ω, item “2)” and in Corollary from sections 0.2.2.1.3.1.1-0.2.2.1.3.1.ω, item “2)” as opposition to the concept “canonical science”. Here the following methodological thesis, significant for their semantic analysis, will be formulated:

The distinction between canonical and interdisciplinary sciences, theories, etc. is essentially connected with the respective distinction between two types of classification of sciences: canonical and interdisciplinary one.

Under the concept “canonical science” I have in mind most generally speaking a science which is element of the so called here canonical classification of sciences, i.e. roughly speaking which is element of the broadly accepted at present scheme of classification of sciences including the ordered set of sciences “mathematics, physics, chemistry, biology...” (the question about exact canonical classification of sciences is not of principal importance at the present stage of our investigation); under the concept “interdisciplinarity” in the field of sciences most generally speaking I have in mind a way by means of which the entire separation and isolation of different sciences as result of their differentiation to be overcome, so, expressing in such a way the integrative tendency in the development of sciences. The interdisciplinarity in the field of sciences can be revealed oneself in various forms among which the following three will be especially mentioned here: **(1)** the interdisciplinary approach to scientific problems – such kind of approach which realise essential connection and even interlacing of elements of different canonical sciences; **(2)** the interdisciplinary investigations – such kind of investigations which realise essential connection and even interlacing of elements of different canonical investigations (i.e. of investigations in the framework of different canonical sciences; **(3)** the interdisciplinary sciences - such kinds of sciences which essentially include in their topic of research problems from different canonical sciences.

Note to section 0.2.2.3.1.2. It is essential that the necessary and sufficient condition for a given science or scientific theory to be interdisciplinary is not merely to deal with problems

of different sciences – such a case is quite usual for canonical sciences, but namely *to include in its topic of investigation* problems from two or more canonical sciences.

0.2.2.3.1.3. Formulation of the preliminary elucidation of meaning of the concept “interdisciplinary approach to foundations of science”

Most generally speaking about interdisciplinary approach to foundations of science I have in mind first of all such an approach to the topic of foundations of science which is based on the treatment of interdisciplinarity as an essential feature of the contemporary science, otherwise speaking which is based on the treatment of the set of interdisciplinary sciences and theories as a fundamental form of existence and development of the science as a whole.

0.2.2.3.2. Towards preliminary explication of meaning of the concept “interdisciplinary approach to foundations of science”

0.2.2.3.2.0. Notes concerning the way of preliminary explication of meaning of the concept “interdisciplinary approach to foundations of science”

In order to achieve appropriate explication of the general intuitive idea of concept “interdisciplinary approach to foundations of science” expressed above in section 0.2.2.3.1 we have to investigate following order of working topics:

- 1) Explication of distinction between canonical and interdisciplinary sciences.
- 2) Scheme of classification of canonical sciences.
- 3) Scheme of classification of interdisciplinary sciences.
- 4) The place of interdisciplinary investigations in the process of historical development of science.
- 5) The interconnection between canonical and interdisciplinary sciences.
- 6) Preliminary explication of the concepts “canonical foundations of science in its integrity” and “interdisciplinary foundations of science in its integrity”.
- 7) Preliminary explication of the concept “interdisciplinary approach to foundations of science”.
- ω) Conclusions from the preliminary explication of the concept “interdisciplinary approach to foundations of science” for the way of obtaining its last explication in the framework of the present introduction.

0.2.2.3.2.1. Explication of the distinction between canonical and interdisciplinary sciences

1) The term “*science of canonical type*”, or merely “*canonical science*”, will be denoted here such a type of science which either is one of the usually adopted at present time to be treated as fundamental science - mathematics, physics, chemistry, biology, psychology, some elements of social sciences like structural linguistics, or is one of their subdivisions as algebra, geometry, mechanics, thermodynamics, quantum fields theory, genetics, physiology, etc. Correspondingly, a scientific task, respectively a scientific problem which is formulated and has to be treated in the framework of a given canonical science will be called here “*canonical scientific task*”, respectively, “*canonical scientific problem*”. In a similar way by the term “*canonical scientific investigation*” will be denoted here such a kind of scientific investigation which in principle can be accomplished in the framework of a given canonical science. Besides, the last one presupposes particularly that the scientific task (respectively the scientific problem) which is subjected to a process of performance in the framework of a given canonical science belongs to (i.e. it is formulated in) the given canonical science. Further, a science or scientific theory will be called here *canonical* one if and only if its problems and the processes of their solution belong to a given canonical science.

2) Sciences (as well as scientific investigations) characterized as interdisciplinary in respect to the canonical ones are by definition ones whose subject of study involves components which are subjects of study of different canonical sciences (or scientific investigations), respectively. Correspondingly, they are devoted to study of such kind of compound problems, which involve as components two or more elementary problems belonging to different canonical sciences (or scientific investigations), respectively. This means particularly that the sciences of interdisciplinary type combine problems, statements and methods of research from different canonical sciences or from different canonical scientific theories, synthesizing them, correspondingly, in new unified problems, in new unified statements as their solutions and in new unified methods of their research in order to obtain their solution. In a similar way the scientific investigations of interdisciplinary type consist of different parts or components which belong to different canonical types of scientific investigations.

0.2.2.3.2.2. Scheme of classification of canonical sciences

The so called by the author of present work canonical classification of scientific problems, which is a basis for classification of canonical sciences, is directed from

problems about abstract structures at the highest point of abstractedness and step by step is going below in a descending line to problems about concrete systems at the lowest point of abstractedness.

On the basis of analysis of various types of objects (in the broad sense of the word, in particular including entities, phenomena, relations, properties, operations) in the reality as well as of various elements and forms of the scientific knowledge, especially of the scientific thinking) I shall postulate here the following scheme towards classification of canonical scientific problems and on its ground the following scheme of classification of canonical sciences:

1) On first (highest) level of abstractedness there are two extreme classes of elements representing various kinds of definiteness of two mutually opposite basic types on the ground of distinction between content and form.

1.1) The set of categories - this is a class of extremely abstract elements of such a kind that each of them represents some basic kind of definiteness with regard to its pure content treated in the most abstract way, i.e. devoid of any concretizing form (in German: grundlegende reininhaltliche Bestimmtheit), so that they (i.e. the elements mentioned above) in their cognitive (epistemic) aspect are the most general ideas about such fundamental things as space, time, movement, action, relation, property, content, form, quality, quantity, necessity, contingency, possibility, impossibility and the like, playing a role of source **(1.1.1)** of construction of exact corresponding concepts of them as elements of the language of science obtained by means of their semantic explication, as well as **(1.1.2)** of elaboration **(1.1.2.1)** of basic general approaches to raising and solving of scientific problems, and, further, **(1.1.2.2)** of basic exact methods of scientific research.

It is worthy of note that the concept “category” allows following two explications: **(1.1.1*)** the concept “object category” (“category_{ob}”) denoting basic kinds of definiteness of objects (again in the broad sense of the last word as mentioned above) with regard to their pure content, and **(1.1.2*)** the concept “conceptual category” (“category_{con}”) denoting the concept of any given category_{ob}, consequently, denoting the concept of corresponding basic kind of definiteness of objects with regard to their pure content; on the other side, if the explication of the concept “category” of the kind “category_{con}” is introduced immediately as it takes place above, then we can define the explication “category_{ob}” by means of using the former one in the following way: every given category_{ob} is by definition a denotation of the denotatum of the correspondingly given category_{con}, besides the concept “category_{con}” in the general sense denotes extensionally the set of all single categories_{con}.

1.2) The set of formal structures, respectively of formal systems (the last term is used here in a more general sense in respect to one used by Hilbert – in particular the requirements of Hilbert’s finitism are not accepted): this is a class of elements of such a kind that each of them represents some basic kind of pure formal definiteness (in German: grundlegende reinformale Bestimmung), which in its cognitive (epistemic) aspect is expressed by means of hierarchical systems of signs, S-modelling in a pure formal way various kinds of inference treated in the most general way as specific relations of acceptance (besides, the truth is regarded as a special form of acceptance).

1.ω. The first class - the class of categories, is subject of study by First Scientific Philosophy and in the same time by First Philosophy of Science (which is regarded here as the first one, i.e. as the First Philosophy of Science, together with its applications, chiefly of methodological character, to treatment of fundamental problems of science and of scientific investigations). On this first step of their elaboration, naturally, they can not be scientific in their form, because the reasoning in their framework in the course of treatment of their problems necessarily distinguishes itself by its essentially intuitive character relying merely on the reason (however the last one is refined under the influence of the exact methods developed and applied in the modern science); besides they play the role of preparation of the basement for construction of strict scientific theories as well as the basement for the accomplishment of the investigations in their framework in a strict scientific way, which has to be realized step by step on the next levels of abstractedness.

The elements of second class, i.e. the pure formal systems (structures) of the kind mentioned above, constitute the content of the Formal Mathematics.

2) The second level of abstractedness is the level of conceptual abstract systems. in their most developed form - they have to be presented as interpreted in a very abstract way formal structures, respectively abstract systems. Generally speaking they are obtainable from formal systems taken from the second class of the first level of abstractedness mentioned above in the item “1.2)”, but interpreted by means of abstract concepts and conceptual structures, deeply rooted **(1)** in the set theory (for the case of pure extensional treatments), or **(2)** in the graph theory, respectively in its infinitesimal generalization (especially for the case of intentional treatments), described in an essentially conceptual way. The basic concepts of this type may be treated as explications of corresponding categories from the first class of the first level of abstractedness mentioned above in item “1.1)”.

The abstract systems as they are described above are of epistemic (cognitive) type. We may speak about their denotatum as contentful (i.e. with regard to the contents; in German: inhaltlich) systems of ontological type.

These abstract structures are subject of study by the so-called here Abstract Conceptual Mathematics (in essence the axiomatic system formulated in Hilbert's "Grundlagen der Geometrie" (cf. [39]) belongs to the last one).

On this level of abstractedness the First Scientific Philosophy as well as the corresponding First Philosophy of Science, mentioned above in the item "1.ω)", obtain their additions by elaborating Second Scientific Philosophy, or, using a term invented by B. Russell (cf. [38]) one may say - by elaborating of Mathematical Philosophy of first order, and together with by elaborating of Second Philosophy of Science, respectively.

3) The third level of abstractedness contains such kinds of structures, respectively of systems, which in their elaborated mathematical form has to be regarded as obtained from structures of the second level of abstractedness, mentioned above in item "2)", but reinterpreted by means of relatively more concrete concepts expressing various kinds of definiteness which are rooted in the nature of concrete things, immediately or indirectly given us in the experience (cf. below item "4)"), however treated in most general and abstract way as pure forms of abstract elements. These kinds of structures are of two basic forms - **(3.1)** abstract ontological structures concerning integrative (and in the same time general) aspects of the entities of physical reality as space, time, processes, interactions, causal connections, etc., and **(3.2)** abstract cognitive (epistemic) structures concerning integrative aspects of the knowledge, especially of the scientific knowledge, as for example, of statements, relations of logical inference between them, etc.

These kinds of abstract structures are subject of study by the so called here Concrete Conceptual Mathematics (the adjective "concrete" is used in a relative sense – the so called above "Concrete Conceptual Mathematics" in its most concrete form is on more abstract level than a natural science elaborated in most abstract form). They express specific ontological or epistemic (cognitive) aspects of mathematical theories and systems but treated in a very general way. In this connection we have in mind first and foremost the following three cases: **(3.1*)** the intuitive visual treatment of the space in geometry, **(3.2*)** the intuitive visual treatment of the casual events and processes in the probability theory, and **(3.3*)** the intuitive visual treatment of the drawing of an inference from some premises in the logic.

The third level of abstractedness is essential at least in the following two aspects:

3.1) It is a connective link (or, using a figurative expression: a bridge) between the very abstract mathematical structures on the first and second levels of abstractedness, mentioned above in items "1)" and "2)", on the one hand, and the concrete systems of reality on the fourth level of abstractedness, which will

be mentioned below in item “4)”, on the other hand. This connection can be realized in following two directions:

3.1.1) The application of mathematical structures related to the first and second levels of abstractedness, as well as of corresponding mathematical methods, to various concrete objects of the fourth level of abstractedness (particularly for their mathematical modelling), realized by means of succession of interpretations from more abstract to immediately more concrete ones passes through the third level of abstractedness.

3.1.2) The process of revealing (or, figuratively speaking: of crystallization) of an abstract structure on the first or second level in its pure form starting from a given system of the fourth level of abstractedness step by step purifying the totality of its concrete characteristics going in such a way to more and more abstract structures, also has to pass (but in the opposite direction) through the third level of abstractedness.

3.2) It is possible to apply mathematical structures of third level to investigations of mathematical structures of first and second levels of abstractedness, i.e. it allows elaboration of Metamathematics.

3.ω. (1) These connections just mentioned above in items “3.1)” and “3.2)” make the second class of the first level of abstractedness and the next two levels of abstractedness to constitute a relatively closed integral system - the system of Mathematics.

(2) On this level of abstractedness the Second Scientific Philosophy as well as the corresponding Second Philosophy of Science, mentioned above in item “2)”, obtain their additions by elaboration of Third Scientific Philosophy, or, again using the term invented by B. Russell (cf. [38]) one may say by elaboration of Mathematical Philosophy of second order, and by elaboration of Third Philosophy of Science, respectively.

4) The fourth level of abstractedness are objects in the broad sense characterized as possessing some or other concrete nature, but treated in a relatively general and abstract way connected with their S-modelling. Taking into account some empirical facts about development of scientific knowledge we have to divide this level on the following three sublevels:

4.1) First sublevel - this is the sublevel of abstractedness involving various kinds of objects in the nature and human society as they are themselves. They are subject of investigations by such kinds of sciences as physics, chemistry, biology, psychology, structural linguistics. On this sublevel has to be elaborated Fourth Scientific Philosophy and correspondingly Fourth Philosophy of Sciences.

4.2) Second sublevel - this is the sublevel of practically oriented scientific knowledge (it will be called here in its totality “praxiomatics”): technology,

agriculture, medicine, management, education. On this sublevel take place the scientific foundations of praxiomatics as well as on it has to be elaborated Fifth Scientific Philosophy and Fifth Philosophy of Sciences.

4.3) Third sublevel - this is the sublevel of things and processes concerning the scientific research. It consists of two basic parts:

4.3.1) History of science

4.3.2) Scientific organization, involving in particular education and management of scientific research.

0.2.2.3.2.3. Scheme of classification of interdisciplinary sciences

Depending on two differential peculiarities of the subject of study of interdisciplinary sciences the last ones are divided into following two types:

First of all the interdisciplinary sciences can be divided into following two types:

- 1) Boundary sciences.
- 2) Complex sciences.

0.2.2.3.2.3.1. Scheme of classification of boundary sciences

1) Interdisciplinary sciences whose subject of study involves objects and phenomena on the boundary between two qualitatively different types of objects and phenomena, correspondingly treated in two adjacent canonical sciences will be called “**boundary sciences**”. In a similar way we can speak about “**boundary problems**”, “**boundary theories**” and “**boundary investigations**”.

As examples for a boundary science we can mention physical chemistry and biochemistry.

2) The set of all names of pairs of adjoining canonical sciences can be treated as the set of indications of all possible boundary sciences. The problem concerning whether the indication of a given possible boundary science, i.e. whether a given pair of adjoining canonical science, is indication of a real boundary science, requires concrete investigations and can obtain its solution only as a result of such kind of concrete investigations.

In a similar way one may say also concerning classification of boundary problems, theories and investigations.

0.2.2.3.2.3.2. Scheme of classification of complex sciences

0.2.2.3.2.3.2.1. Interdisciplinary sciences **(1)** whose subject of study involves different parts and aspects which are subject of study of different

canonical sciences (2) besides they are not boundary ones, (1→2) will be called “**complex sciences**”.

Correspondingly, the scientific problems involving as subproblems a variety of more elementary problems belonging to different canonical sciences besides they are not boundary problems will be called “**complex scientific problems**”. Analogically we can speak about “**complex scientific investigations**” and “**complex scientific theories**”.

And so, generally speaking *complex sciences* are ones (1) whose *subject of study* are *complex objects*, i.e. such kind of objects which involve as components subjects of study of various different canonical sciences, (2) in which *complex scientific problems of study* are raised, and (3) for their solutions *complex scientific investigations* are used.

0.2.2.3.2.3.2.2. In addition to just proposed description of the sense of term “complex science” we have to specify following two basic forms of complex sciences:

1) *Complex sciences of first order* - these are complex sciences based on abstract objects and correspondingly on abstract elements of knowledge. More detailed speaking the complex objects which serve as subject of research in this case are abstract structures (respectively, abstract systems) of first, second or third level (cf. section 0.2.2.3.2.2, item “1”)–“3”) together with the variety of their successive more and more concrete interpretations belonging to different canonical sciences.

Note to section 0.2.2.3.2.3.2.2, item “1”). On the basis of the classification of levels of abstractedness of the structures, mentioned in section 0.2.2.3.2.2, we shall postulate the following **canonical structure of the complex sciences**:

1) On first place – elaboration (1) of foundations of the language of given field of science or of given scientific theory and (2) of the general type of approach to raising and way of solution of its tasks (in particular: problems) (1→2) which is deeply rooted in the philosophical analysis of a *group of categories* treated as fundamental semantic units of this language - level of abstractedness 1.1;

2) On second place – elaboration of a hierarchy of mathematical structures which in their most developed form consist of (1) pure formal systems (syntax) - level of abstractedness 1.2, and (2) systematic interpretation of the last ones by means of abstract structures which are deeply rooted (2.1) in the set theory (for the case of pure extensional treatments), or (2.2) in the graph theory, respectively in its infinitesimal generalization (especially for the case of intentional treatments) - level of abstractedness 2, besides (3) the structures of level of abstractedness 2 can be further reinterpreted by means of structures of the level of abstractedness 3, which are of two forms - (3.1) general ontological structures concerning integral (and in the same time general) aspects of the reality as space, time, process, etc., which represents a system of explications of corresponding object categories - level of abstractedness 3.1, and (3.2) abstract cognitive structures concerning integral (and in the same time general) aspects of the knowledge, especially of the scientific knowledge, as for example concepts, statements, logical inference, etc., which

represents a system of explications of corresponding subject (epistemic) categories - level of abstractedness 3.2.

3) On third place – elaboration of structures of concrete entities and phenomena - level of abstractedness 4.

4) On fourth place – elaboration of structure of pragmatic systems and processes: technology, medicine and so on.

5) On fifth place – elaboration of conceptual history of the given field of science and future perspectives.

2) *Complex sciences of concrete objects (they may be called also: complex sciences of second order)* - they have to deal with all-embracing investigation of a given concrete object or type of objects distinguished as very complicated so that the study of various its parts and aspects turn out to be subject of study of various canonical sciences (as for example the earth which as very complicated object in the sense just described is subject of study of the complex science of second order called “science of earth”).

0.2.2.3.2.4. The place of interdisciplinary investigations in the historical development of science

0.2.2.3.2.4.1. The essence of the division into periods of the history of science consists in revealing of hierarchical temporal structure, i.e. of revealing of boundaries between historical units of time of various levels (the first three of which are called here respectively "periods", "stages", "phases") together with their specifications - of the process of historical development of science according to the real facts of this process.

0.2.2.3.2.4.2. The criteria for the division into periods of the history of science, which have to ensure its objective value, concern in particular the following:

- 1) the scopes of scientific inquiries during the historical units of time;
- 2) hierarchical structure of scientific knowledge expressing the relationship between its parts (fields, branches, etc.) and essentially connected with the interrelations between tendencies of its differentiation and integration;
- 3) the interconnections of science with other spheres of the culture, in particular with technology and medicine.

0.2.2.3.2.4.3. A scheme of division into periods of the history of science

0. Period of the Prehistory of Science: from the origin of Homo Sapiens to Thales (7th c. BC). The practical knowledge is developed to an increasing degree and in various directions, so, in the framework of this development a process of creating the necessary prerequisites for formation of the first

elements of scientific knowledge as well as of a rational philosophy closed connected with last ones takes place (cf. section 0.2.1, fact 2).

I. *First Period of the History of Science:* from Thales (7th c. BC) to the beginning of Renaissance (about the middle of the 15th c.). The integrative tendency in the process of development of theoretical (scientific or semiscientific) knowledge has been dominated during this period. This circumstance has been expressed by means of domination of the philosophy as an almost all-embracing theoretically systematized knowledge, involving some scientific hypotheses, or even some elements of scientific knowledge in an embryonic form, besides in the same time the mathematics and medicine obtained independent development. (cf. section 0.2.1, facts 1, 3-13)

II. *Second Period of the History of Science:* from the beginning of Renaissance (about the middle of 15th c.) to about the middle of the 20th c. During this period it takes place **(1)** a process of formation and further development of natural sciences, involving **(1.1)** a process of invention, production and application of various kinds of means of exact observations, measurement and experimentation of corresponding kinds of objects and phenomena, as well as **(1.2)** a process of use to an increasing but different for various branches degree of mathematical methods, besides, in some cases it takes the form of mathematical modelling; **(2)** a process of application to an increasing degree of various branches of natural sciences and mathematics (for the last one: in a direct form, or by means of its use to the natural sciences mentioned above in item (1.2)) to the various spheres of praxis, particularly to the field of the technology, medicine and agriculture; besides this circumstance has being reflected back upon their development stimulating it considerably; **(3)** domination of the process of differentiation of the science to an increasing degree (which does not exclude however in some degree also processes of integration). (cf. section 0.2.1, facts 14-16).

III. *Third Period of the History of Science:* since the middle of 20th c. or more exactly speaking since N. Wiener's "Cybernetics" - its acceptance as a historical unit of the process of development of the science is founded partially on historico-empirical facts and in the same time partially it has the character of a philosophico-methodological hypothesis. This period is characterized on the first place by the essential role of the interdisciplinary investigations in science resulting in formation of complex theories, which are not sciences of Newtonian type - they represent a new type of synthesis of scientific problems, unifying in the same time problems and results taken from essentially various theories of Newtonian type. The first stage of this period, which is still lasting, is characterized by the process of increasing intensification of the role of

interdisciplinary investigations resulting in formation and further elaboration of various complex sciences. (cf. section 0.2.1, fact 17).

0.2.2.3.2.4.4. The place and the role of interdisciplinary sciences and investigations in modern science

1) In accordance with the scheme of division into periods of the history of science formulated in previous section 0.2.2.3.2.4.3 the formation and further development of interdisciplinary sciences and investigations at the present time are not merely one of the peculiarities of the last one, but moreover, it plays a deeper and a more profound role in the history of science – these processes mark the beginning a new period in the historical development of science. In particular, one of the sequences of this statement is that the interdisciplinary approaches and investigations in the contemporary science, opening a new period in the history of science, require also a new philosophy of science as well as a new methodology of the researches in the field of history of science - they require interdisciplinary approach to the history and philosophy of science!

2) The interdisciplinary approaches and interdisciplinary investigations in the modern science are expressions of the integrative tendency, which was already mentioned in the characteristic of the first period of historical development of science as dominated during that time (cf. section 0.2.2.3.2.4.3, item “I.”). So, having in mind this fact we can speak about a return during the historical process of development of science back to characteristic feature of the first period: domination of integrative tendency. However in the same time we have to pay attention to the fact, that this return is essentially conditional – there is principle difference between integrative tendency in the scientific knowledge during ancient time and during modern time: **(1)** the integrative tendency in the scientific knowledge during ancient time was something immediate and primary, whereas **(2)** the integrative tendency in modern science presupposes the differentiation in science during second period of its history and it is accomplished namely on the basis of this very developed differentiation in science. So, the emphasized by Hegel dialectical “negation of the negation” takes place in this case.

0.2.2.3.2.5. The interconnection between canonical and interdisciplinary sciences.

The elaboration of both types of sciences – canonical and interdisciplinary ones, is essential for the contemporary process of historical development of science, besides these two types are in mutual interaction: the development of canonical sciences is inevitable condition for formation as well as factor for

further development of interdisciplinary sciences on the one hand and the development of interdisciplinary sciences enriches the various canonical sciences with new concepts, problems, statements and methods of research. Consequently neither the formation and development of interdisciplinary sciences is rejection of the canonical sciences, nor the development of canonical sciences is rejection of the interdisciplinary sciences, on the contrary, their progressive development is in mutual interconnection.

**0.2.2.3.2.6. Preliminary explication of the concepts
“canonical foundations of science as a whole” and
“interdisciplinary foundations of science as a whole”**

0.2.2.3.2.6.1. In accordance with distinction between canonical and interdisciplinary types of sciences we have correspondingly to distinguish between two basic types of foundations of science as a whole:

1) Canonical foundations of science as a whole means a system of successive foundations of various fields of canonical sciences in the order corresponding to their canonical classification – foundations of mathematics, foundations of physics, etc. which (in accordance with section 0.2.2.2.1) consist in **(1)** complete systematic theoretical formulation of their peculiar (for every field of science) foundational kernel, and **(2)** demonstration that starting from such a theoretical formulation of the given foundational kernel and carrying out the corresponding procedures according to it we can (eventually with the help of additional considerations) obtain the multiformity of propositions making up the content of the corresponding field of science (usually it is enough to obtain in such a way the smaller set of the most important its characteristic propositions).

Besides at present state of development of science it is still an open question whether **(1*)** mathematics and physics may be considered to be really unified sciences which acceptance presupposes the availability of united foundational kernel in their frameworks or **(2*)** they are also ordered sets of separate branches of mathematics or physics, correspondingly, with specific foundational kernels for each of them. Moreover I assume the possibility to speak about different mathematics with correspondingly different foundational kernels in accordance with different principal positions in the philosophy of mathematics as logicism, formalism, constructivism, etc.

In this connection we can formulate the following *methodological hypothesis* concerning the content of the foundational kernel of the various kinds of canonical sciences: **(1**)** the foundational kernel of mathematics and mathematical (respectively: deductive) sciences consists of the relations of

cognitive determination mentioned in item (3.1) of section 0.2.2.1.3.2.2.4; (2**) the foundational kernel of natural (respectively: ultimately inductive) sciences consists of some kind of causal relationship in the corresponding subject of research, producing the objective unity in the enormous variety of different phenomena mentioned in the item (3.2) of section 0.2.2.1.3.2.2.4. This hypothesis has found confirmation in the facts concerning formation of sciences stated in section 0.2.1; in the same time some rational considerations about the different nature of these two spheres of canonical sciences corroborate it.

2) Speaking about **interdisciplinary foundations of science as a whole** we shall have in mind first of all **foundations of the complex sciences in their totality**.

By analogy with the case concerning the canonical sciences mentioned above in item “1)” we can formulate the following *methodological hypothesis* concerning the content of the foundational kernel of the various kinds of complex sciences:

2.1) The complex sciences of first order (i.e. which are based on abstract objects and abstract elements of knowledge) allow availability of a general unified foundational kernel consisting of the relations of cognitive determination mentioned in item (3.1) of section 0.2.2.1.3.2.2.4, so that its complete systematic theoretical exposition plays the role of general foundations of the complex sciences of first order which serves as common basis for elaboration of various more special complex sciences and scientific theories of first order.

2.2) The complex sciences of second order (i.e. which are of concrete objects as for example of earth) does not allow availability of a general unified foundational kernel, besides the foundational kernel of each consists in some kind of causal relationship in the corresponding subject of research, producing the objective unity in the enormous variety of different phenomena, mentioned in the item (3.2) of section 0.2.2.1.3.2.2.4.

0.2.2.3.2.6.2. Here I propose the following methodological hypotheses concerning the general unified foundational kernel of the complex sciences of first order: it consists of general theoretical exposition of the three types of relations of cognitive determination mentioned in item (3.1) of section 0.2.2.1.3.2.2.4:

- 1) Methodology of S-modelling (including as a basis the methodological treatment of S-models)
- 2) Logic of science
- 3) Theory of categories.

Taking into account the interconnection between *the Methodology of S-modelling* and *the Logic of science* it is reasonably to accept their joint treatment as unified science called Logistics. So, *the general foundations of complex sciences of first order may be considered the Logistics according the stipulation that its philosophic-methodological basis is a theory of categories.*

0.2.2.3.2.6.3. Owing to the treatment of categories generally speaking as forms of definiteness/indefiniteness, the first step of their systematic analysis consists in distinction between definiteness and indefiniteness. Correspondingly the Logistics in its analytic form has to be divided into following two parts:

- 1) Definitics (complex theory of definite structures, aggregates, processes and systems, treated on various levels of abstractness and generality),
- 2) Indefinitics (complex theory of indefinite structures, aggregates, processes and systems, treated on various levels of abstractness and generality).

Note to section 0.2.2.3.2.6.3. It is quite natural after elaboration of the interdisciplinary approach to foundations of science in the form of analytical logistics, as it is characterized in the present section 0.2.2.3.2.6.3, possessing a significant heuristic role for the research, the exposition of the last one to be supplemented with construction of logistics in an synthetic form in which the pure formal fragments and the corresponding conceptual (contentful) fragments to alternate in the usual way.

0.2.2.3.2.7. Preliminary explication of the concept “interdisciplinary approach to foundations of science”

Taking into account (1) the formulation of the preliminary elucidation of meaning of the concept “interdisciplinary approach to foundations of science” in section 0.2.2.3.1.3, as well as (2) the distinction in the previous section 0.2.2.3.2.6 between two types of foundations of science as a whole – (2.1) foundations of the canonical sciences in their totality, and (2.2) foundations of the interdisciplinary (in particular: of the complex) sciences in their totality (1) → (2) we shall formulate the following thesis concerning the preliminary explication of the concept “interdisciplinary approach to foundations of science”:

Thesis about the scope of the concept “interdisciplinary approach to foundations of science”. Taking into account what was mentioned in section 0.2.2.3.2.6.2 the general approach to foundations of the complex sciences of first order turns out to be an approach to elaboration of logistics according the stipulation that its philosophic-methodological basis is a theory of categories. Besides the so treated logistics plays an essential nevertheless additional, secondary role for foundations of complex theories (sciences) of second order as well as for so called here boundary sciences too.

In view of this fact one may say that the approach to elaboration of logistics (according the stipulation that its philosophic-methodological basis is a theory of categories) is namely the general approach in an broad sense to foundations of interdisciplinary sciences.

0.2.2.3.2.ω. Conclusions from the preliminary explication of the concept “interdisciplinary approach to foundations of science” for the way of elaboration of interdisciplinary approach to foundations of science

Thesis 1. Taking into account what was mentioned in section 0.2.2.3.2.5 concerning the interconnection between canonical and interdisciplinary sciences we shall accept that the process of elaboration of the interdisciplinary approach to foundations of science has to be preceded from specification of the canonical approach to foundations of science.

Note to section 0.2.2.3.2.ω, Thesis 1. The specification of the canonical approach to foundations of science mentioned in the thesis 1 plays the role of addition to the conditions in the posing the task of present investigation (cf. Note 2 to section 0.1.1, item “2”).

Thesis 2. Taking into account that the historical process of differentiation of sciences passes to a considerable extent spontaneously we shall accept that for the purposes of elaboration of the interdisciplinary approach to foundations of science it is needed the specification of the canonical approach to foundations of science mentioned in the thesis 1 to take the form of elaboration of integrative rational differentiation of canonical sciences, i.e. of such one which is distinguished itself by critical reconstruction of the classification of the canonical sciences by means of successive systematic division of the science as a whole step by step following the direction from more abstract, more formal, more general to more concrete, more contentfull, to the singular.

Note to section 0.2.2.3.2.ω, Thesis 2. The reason for acceptance of thesis 2 consists in the circumstance that the integrative rational approach to differentiation of the canonical sciences reveals the systematic relations between them and in such a way it contributes the revealing the interdisciplinary connections between them and so it contributes formation and further development of interdisciplinary sciences.

Note to the Introduction. The specification of the canonical approach to foundations of science mentioned in the theses 1 and 2 and the subsequent explication of the concept “interdisciplinary approach to foundations of science” will be formulated in a next article.

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